VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of General Engineering Sciences and Labour Safety

TRANSIENTS IN THE SIMPLEST ELECTRICAL CIRCUITS

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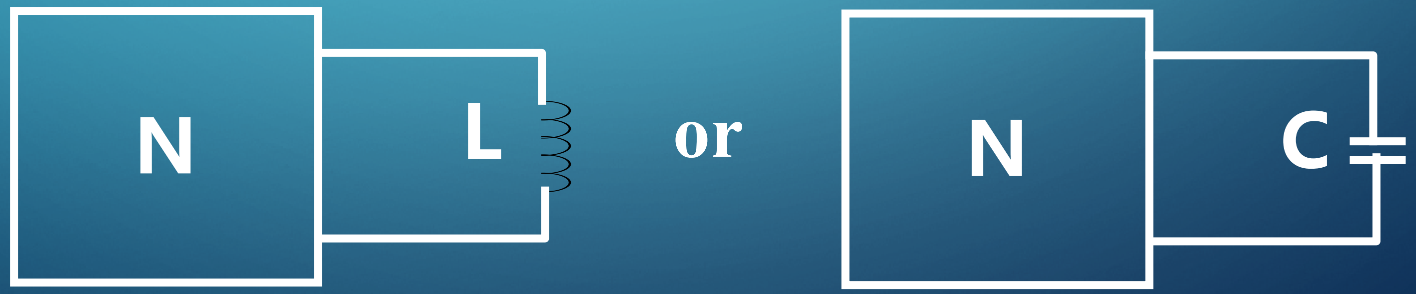
FIRST-ORDER RC CIRCUITS

- **First-order circuit**

  Only one (equivalent) capacitor or inductor is included in a linear circuit.

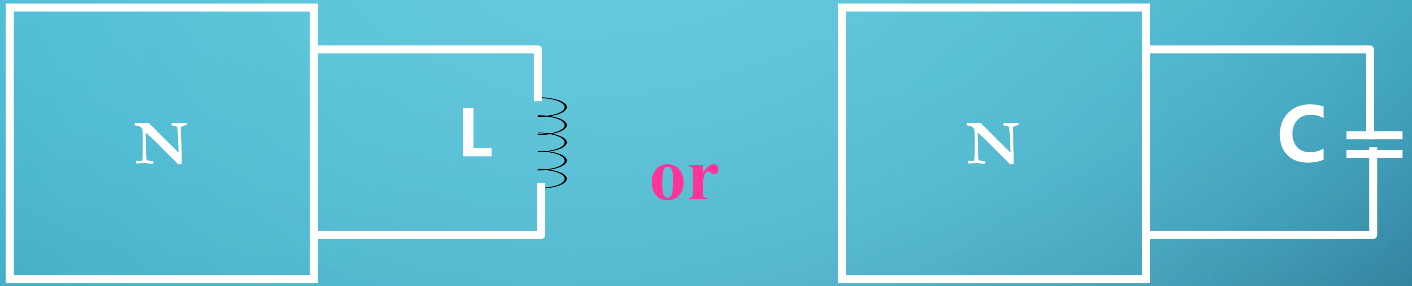
- **Equivalent circuit of First-order circuit**

  Two parts: one (equivalent) capacitor or inductor; a two terminal network with resistance and sources.

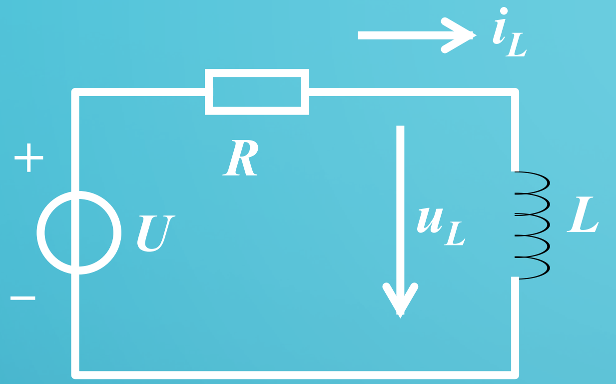


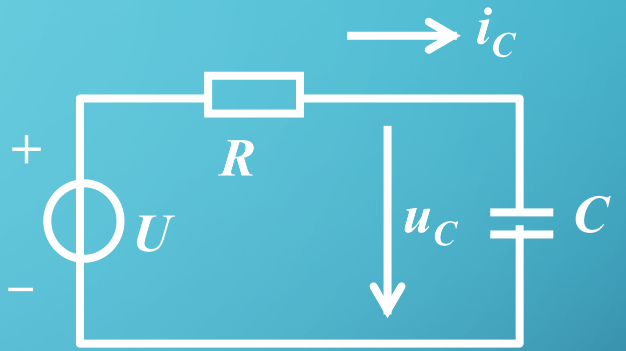
First-order RC Circuits

- **According to Thevenin Law**



Differential equation of first-order RC circuit





$$u_R + u_L = U$$

$$Ri_L(t) + L\frac{di_L(t)}{dt} = U$$

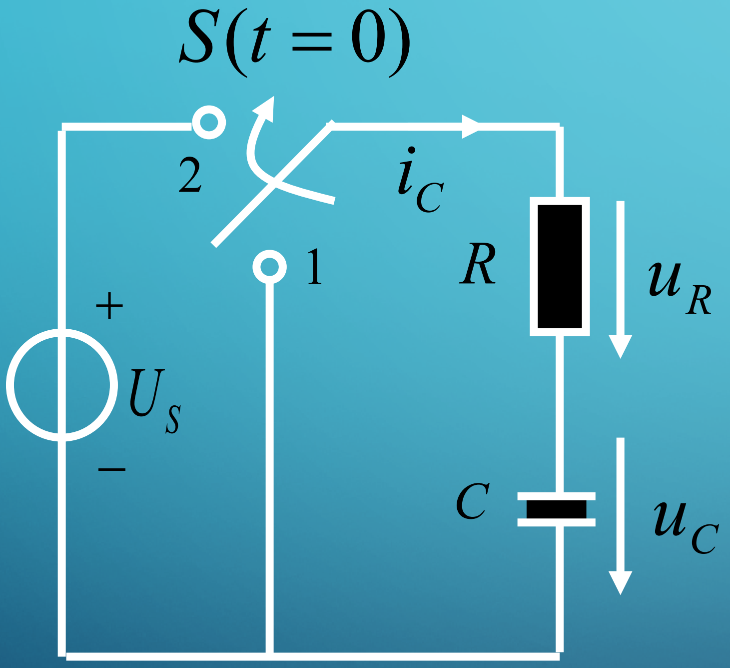
$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = \frac{U}{R}$$

$$u_R + u_C = U$$

$$RC\frac{du_C}{dt} + u_C = U$$

FIRST-ORDER RC CIRCUITS

- **Example: to find the transient response after changing circuit when** t=0.

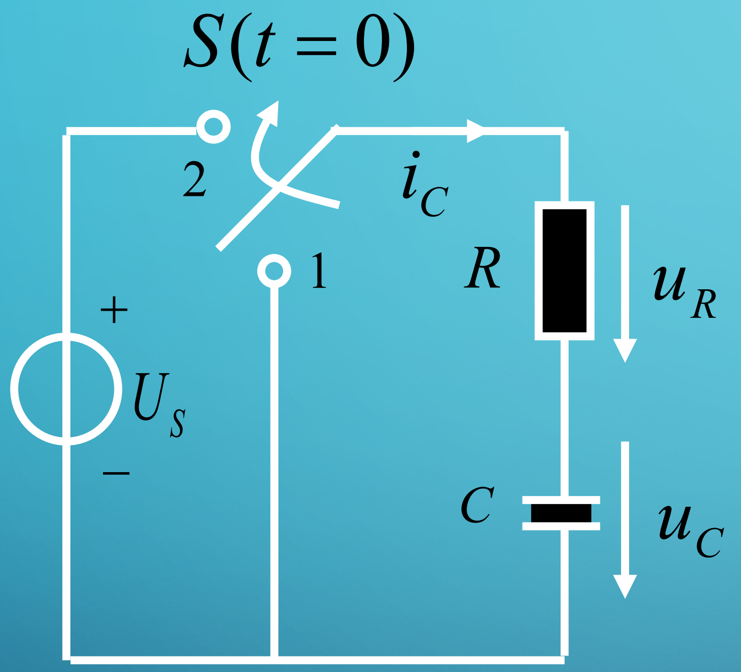


$$u_C(0-)=0$$

Solution:

t \ $f(t)$	u_C	u_R	i
0−	0	0	0
0+	0	U_S	$\frac{U_S}{R}$
∞	U_S	0	0

First-order RC Circuits



$$u_R + u_C = U_S$$

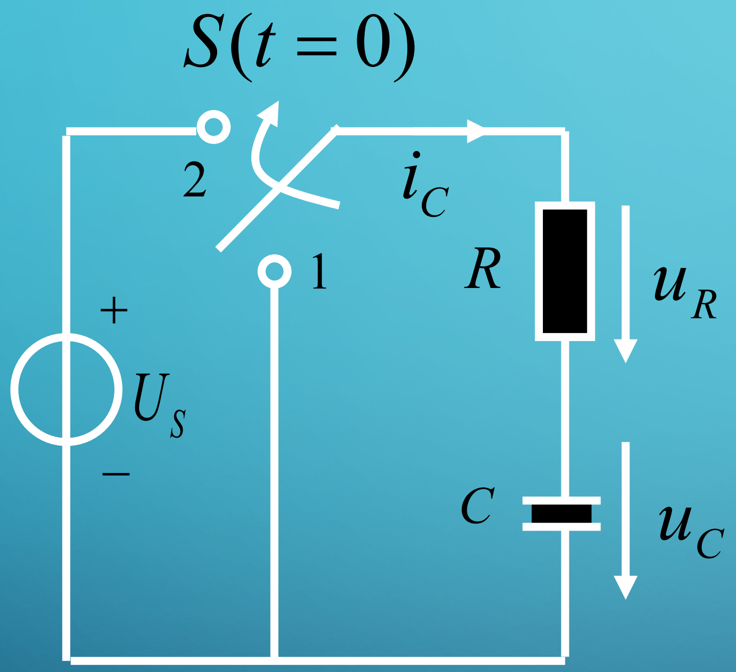
$$u_R = Ri \qquad i = C\frac{du_C}{dt}$$

$$RC\frac{du_C}{dt} + u_C = U_S$$

$$u_C(0-) = 0$$

$$u_C(0+) = u_C(0-) = 0$$

First-order RC Circuits



$$RC\frac{du_C}{dt} + u_C = U_S$$

$$u_C = u_C^{'} + u_C^{''}$$

$$u_C^{'} = Ae^{st}$$

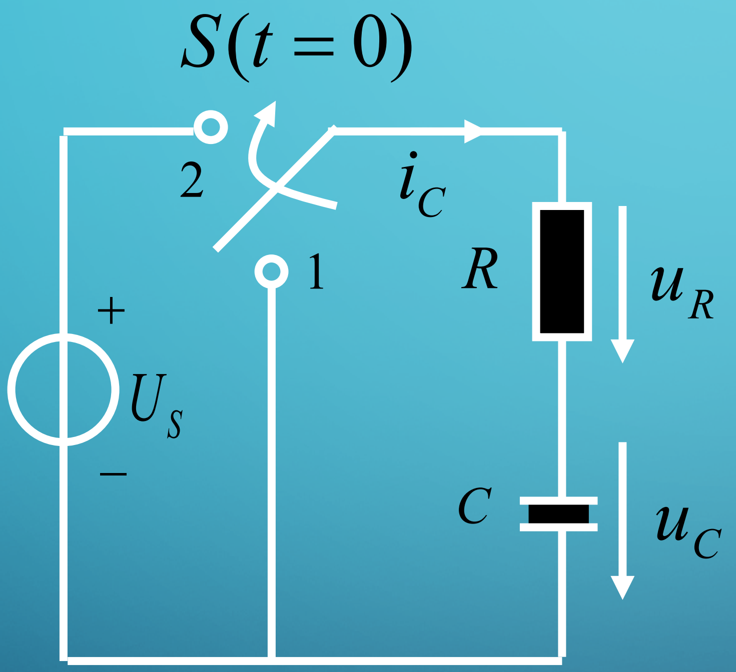
——homogeneous solution

$$u_C^{''}$$

——particular solution

First-order RC Circuits

- **homogeneous solution**



$$RC\frac{du_C}{dt}+u_C=U_S$$

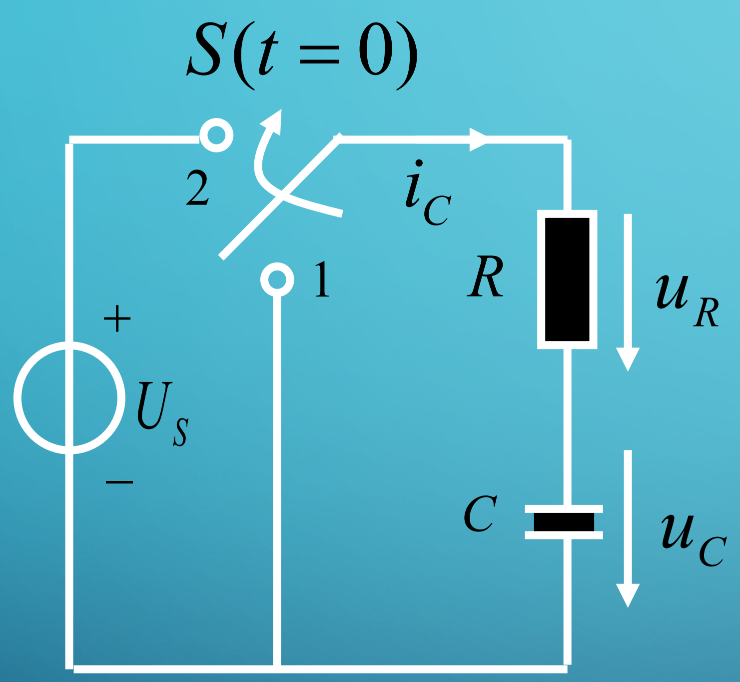
$$RCs+1=0$$

$$s=-\frac{1}{RC}$$

$$u'_C=Ae^{-\frac{1}{RC}t}$$

First-order RC Circuits

- **Particular solution**



$$RC\frac{du_C}{dt}+u_C=U_S$$

Therefore

$$u_C^{''}=u_C(\infty)=U_S$$

Then, the final solution is

$$u_C=u_C^{'}+u_C^{''}=Ae^{st}+U_S$$

First-order RC Circuits

- **The solution of differential equation**

$$u_C = u_C' + u_C'' = Ae^{st} + U_S$$

Substituting the initial condition：

$$u_C(0+) = u_C' + u_C'' = Ae^{s0} + U_S = 0$$

$$A = u_C(0+) - u_C(\infty) = -U_S$$

$$u_C(t) = u_C(\infty) + [u_C(0+) - u_C(\infty)]e^{-\frac{1}{RC}t}$$

$$= U_S - U_S e^{-\frac{1}{RC}t}$$

First-order RC Circuits

- **The solution of differential equation**

$$\tau = RC$$ ——Time constant

$$u_C(t) = u_C(\infty) + [u_C(0+) - u_C(\infty)]e^{-\frac{t}{\tau}}$$

$$u_C(\infty)$$ ——Steady state value

$$u_C(0+)$$ ——Initial value

First-order RL Circuits

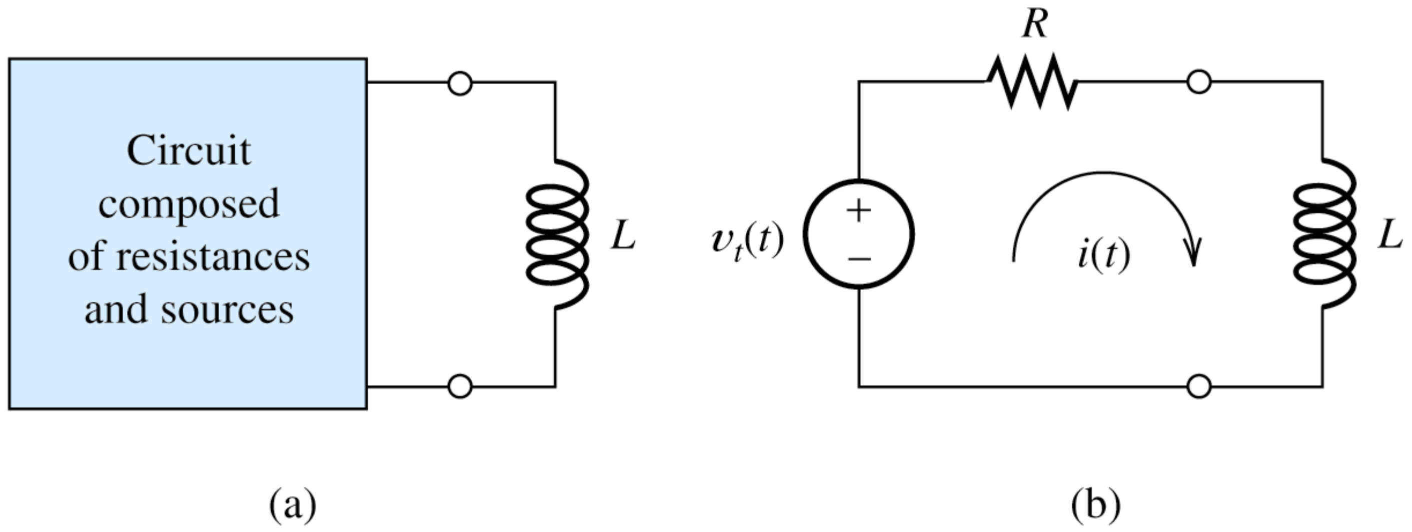
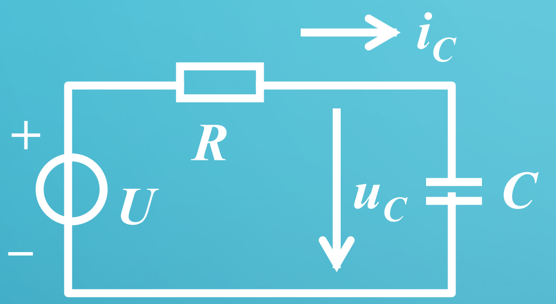


Figure 4.13 A circuit consisting of sources, resistances, and one inductance has an equivalent circuit consisting of a voltage source and a resistance in series with the inductance.

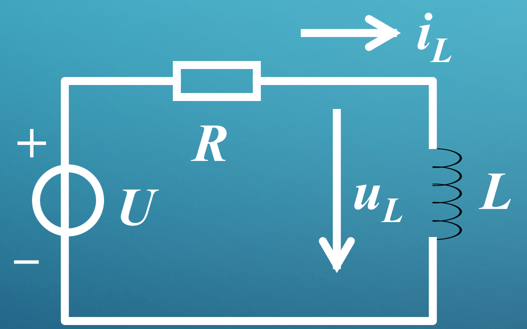
First-order RL Circuits

- **Time constant**



$\tau = RC$

$$RC\frac{du_C}{dt} + u_C = U$$



$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = \frac{U}{R}$$

$\tau = L/R$

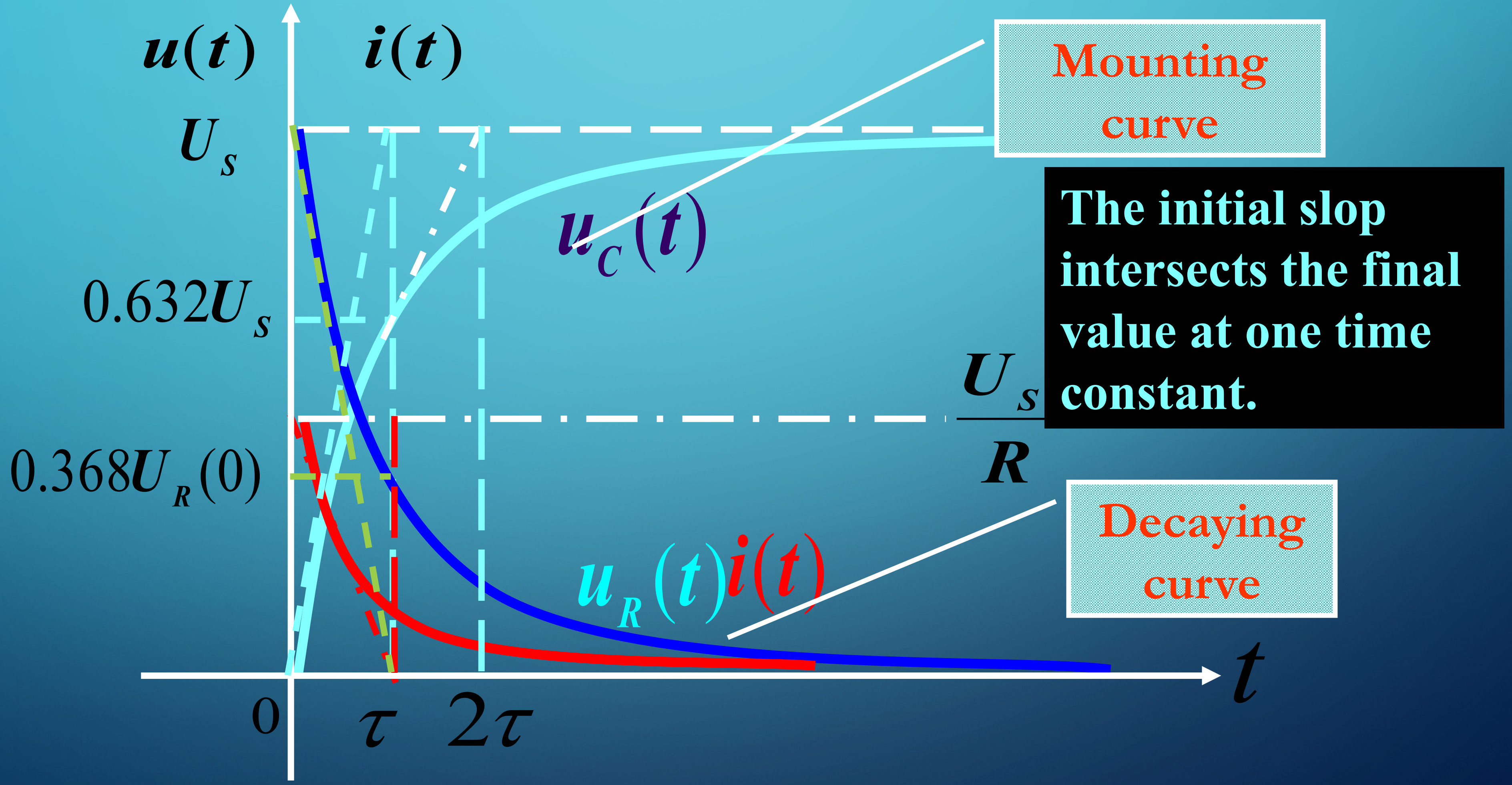
- **Time constant reflects the length of transient period.**

t	τ	2τ	3τ	4τ	5τ	6τ	7τ
$e^{-t/\tau}$	36.8%	13.5%	5%	1.8%	0.3%	0.25%	0.09%

- **After one time constants, the transient response is equal to 36.8 percent of its initial value.**

- **After about five time constants, the transient response is over.**

- **Time constant reflects the length of transient period.**
  - The curves versus time



•Example 4.2 Find voltage of *v*(*t*) and current *i*(*t*) in this circuit for *t*>0.

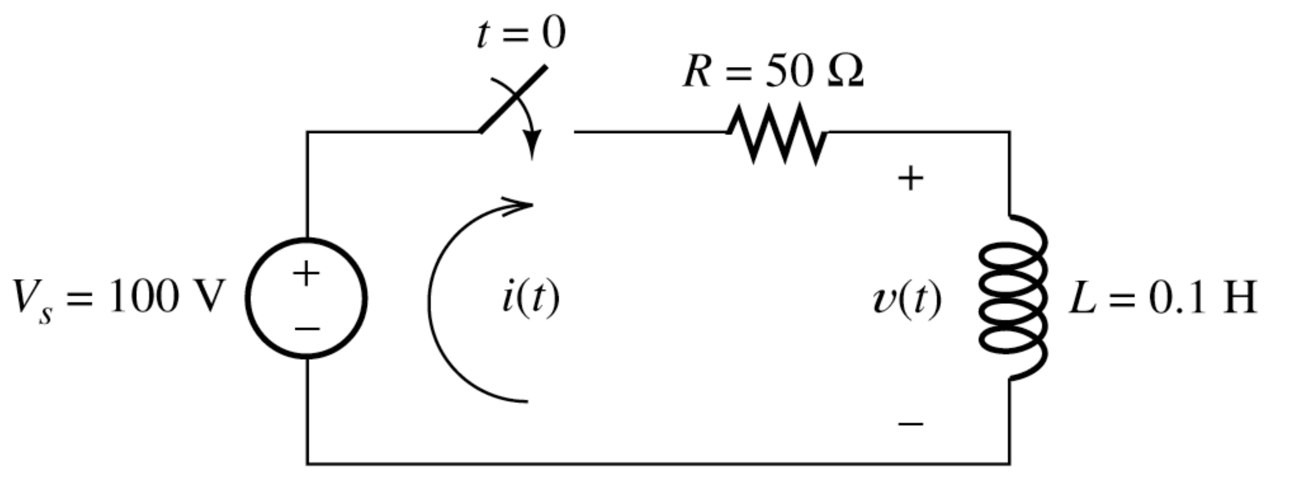


Figure 4.7 The circuit analyzed in Example 4.2.

Answer:

$$i(t) = 2 - 2e^{-\frac{t}{\tau}}(\text{A}),\ v(t) = 100e^{-\frac{t}{\tau}}(\text{V})$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 2(\text{ms})$$

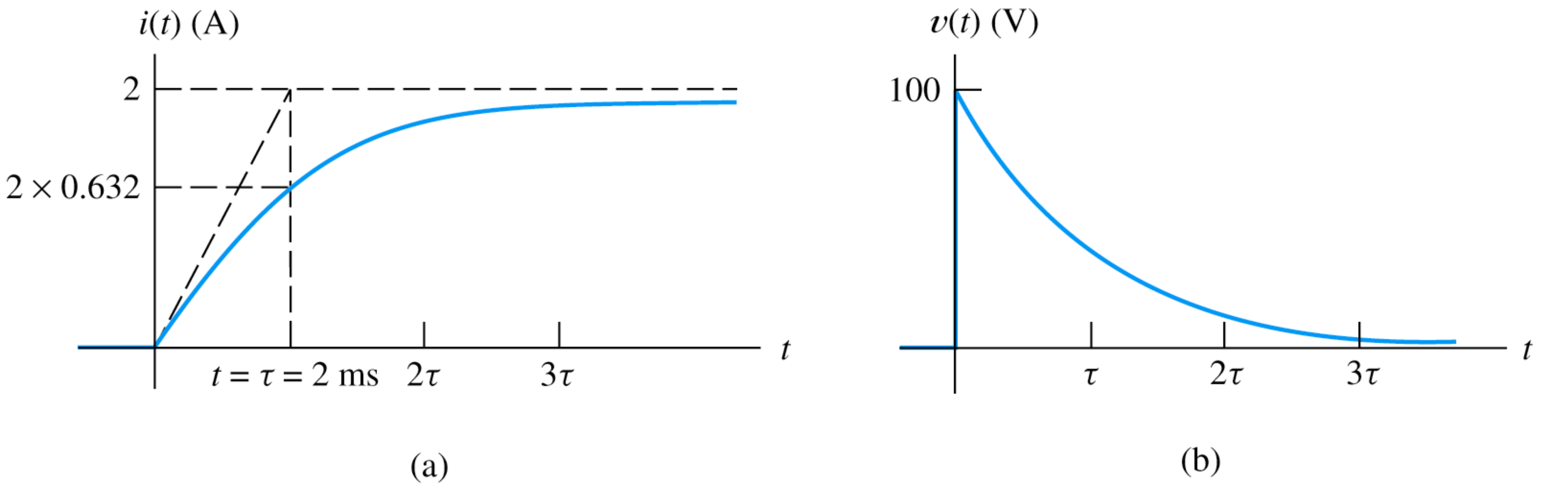


Figure 4.8 Current and voltage versus time for the circuit of Figure 4.7.

$$i(t) = 2 - 2e^{-\frac{t}{\tau}}(\text{A}), v(t) = 100e^{-\frac{t}{\tau}}(\text{V})$$

$$\tau = \frac{L}{R} = \frac{0.1}{50} = 2(\text{ms})$$