VINNITSA NATIONAL AGRARIAN UNIVERSITY

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CALCULATION OF CURRENTS IN NON-SINUSOIDAL CIRCUITS

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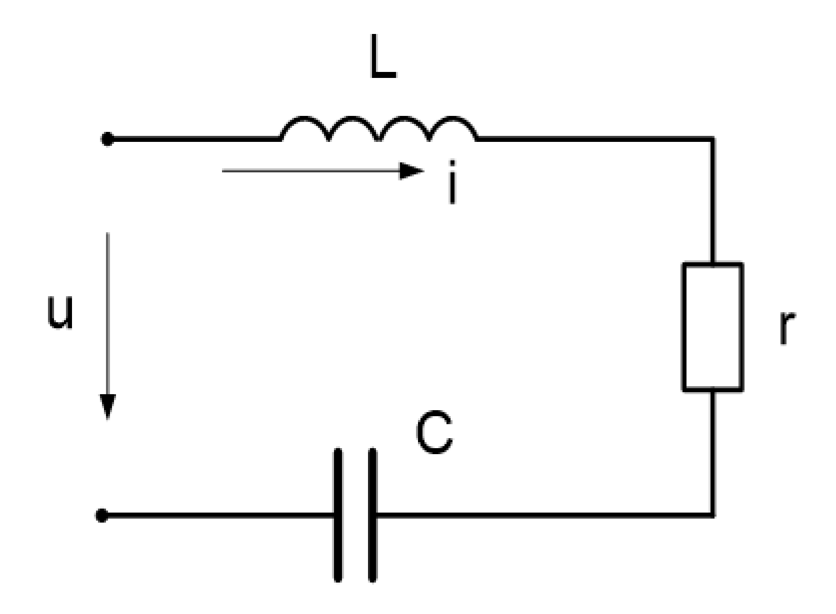
- Using the superimposition theorem, we can say that the currents in the branches can be defined as the algebraic sum of currents from the action of each source separately. So, the calculation of the electric circuit is conducted for each harmonic separately. Since in this case the voltage of the sources is sinusoidal, the complex method can be used, and because the frequencies of the harmonics are different, the resistance of the circuit branches for each harmonic may also be different.

Consider the method of calculation on the example of a simple electric circuit



Let voltage be applied to this circuit

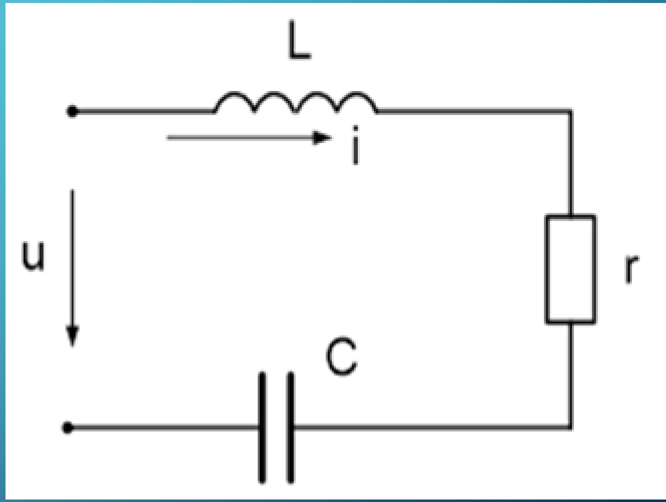
$$u = U_0 + U_{m1}\sin(\omega t + \beta_1) + U_{m2}\sin(2\omega t + \beta_2) + \dots .$$

The zero harmonic of the current in the specified circuit is absent, as the resistance of the capacitor at constant voltage is infinitely large

Current of the first harmonic

$$\underline{U}_{m1} = U_{m1} e^{j\beta_1}.$$

$$\underline{I}_{m1} = \frac{\underline{U}_{m1}}{r + j\left(\omega L - \frac{1}{\omega c}\right)},$$



For the k-th harmonic, the resistance of inductance and capacity will change

$$x_{Lk} = k\omega L,$$

$$x_{ck} = \frac{1}{k\omega c}$$

therefore, the complex amplitude for the harmonic will be equal

$$\underline{I}_{mk} = \frac{\underline{U}_{mk}}{r + j\left(k\omega L - \frac{1}{k\omega c}\right)}, \qquad \underline{U}_{mk} = U_{mk} e^{j\beta k}.$$

The total instantaneous current value will be calculated as the sum of the instantaneous current values in the different harmonics

if $\underline{I}_{mk} = I_{mk} e^{j\alpha k}$ then

$$i = I_{m1}\sin(\omega t + \alpha_1) + I_{m2}\sin(2\omega t + \alpha_2) + I_{m3}\sin(3\omega t + \alpha_3) + \ldots \quad .$$

Any calculation method can be used to calculate the current of each harmonic

Effective value of voltage and current

The effective value of non-sinusoidal currents and voltages is found as the square root of the sum of squares of the effective values of individual harmonics

$$I = \sqrt{I_0^2 + I_1^2 + I_2^2 + I_3^2 + \dots}$$

$$U = \sqrt{U_0^2 + U_1^2 + U_2^2 + U_3^2 + \dots}$$

When using amplitude values

$$I = \sqrt{I_0^2 + \frac{I_{m1}^2}{2} + \frac{I_{m2}^2}{2} + \frac{I_{m3}^2}{2} + \dots}$$

Power in electrical circuits with non-sinusoidal currents

$$P = \frac{1}{T}\int_0^T p dt,$$

where p - instant power

$$p = ui$$

If

$$u = U_0 + U_{m1}\sin(\omega t + \beta_1) + U_{m2}\sin(2\omega t + \beta_2)\ldots,$$

$$i = I_0 + I_{m1}\sin(\omega t + \alpha_1) + I_{m2}\sin(2\omega t + \alpha_2)\ldots,$$

then

$$P = \frac{1}{T}\int_0^T [U_0 + U_{m1}\sin(\omega t + \beta_1) + U_{m2}\sin(2\omega t + \beta_2) + \ldots] \times$$

$$\times [I_0 + I_{m1}\sin(\omega t + \alpha_1) + I_{m2}\sin(2\omega t + \alpha_2) + \ldots] dt.$$

After the implementation of mathematical transformations, we obtain

$$\int_0^T U_0 I_0 dt = U_0 I_0 T,$$

$$\int_0^T U_0 I_{mk} \sin(k\omega t + \alpha_k) dt = 0,$$

$$\int_0^T I_0 U_{mk} \sin(k\omega t + \beta_k) dt = 0,$$

$$\int_0^T U_{mk} \sin(k\omega t + \beta_k) I_{mk} \sin(k\omega t + \alpha_k) dt = \frac{U_{mk} I_{mk}}{2} \cos\varphi_k,$$

From here

$$P = U_0 I_0 + U_1 I_1 \cos\varphi_1 + U_2 I_2 \cos\varphi_2 + U_3 I_3 \cos\varphi_3 + \ldots \quad .$$

The active power in circles with non-sinusoidal current is equal to the sum of the powers in each of the harmonics

$$P = P_0 + P_1 + P_2 + \cdots..$$

The reactive power in circles with non-sinusoidal current is equal to the sum of the powers in each of the harmonics

$$Q = Q_1 + Q_2 + Q_3 + \cdots..$$

By analogy with sinusoidal circles, we introduce the concept of full power

S = U I

where

$$I = \sqrt{I_0^2 + I_1^2 + I_2^2 + I_3^2 + \ldots} \qquad U = \sqrt{U_0^2 + U_1^2 + U_2^2 + U_3^2 + \ldots}$$

It should be remember that in circles with non-sinusoidal currents and voltages

$$S^2 \neq P^2 + Q^2$$

Coefficients that characterize the difference between the curve and the sine wave

To assess the difference between the shape of a non-sinusoidal signal from the sinusoidal, different coefficients (factors) are introduced.

Amplitude coefficient

$$K_a = \frac{I_m}{I} = \frac{I_m}{\sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots}} .$$

Distortion coefficient

$$K_c = \frac{I_1}{I} = \frac{I_1}{\sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots}}$$

Coefficient of nonlinear distortion

$$K_{н.с} = \frac{\sqrt{I_2^2 + I_3^2 + I_4^2 + \ldots}}{I_1}$$

EXAMPLE OF CALCULATION (individual work)

Calculate the current in the circuit

$$u(t) = 200 + 290\sin(\omega t) + 195\sin(2\omega t + 20^0)\ V$$

$$R = 10\ Ohm$$

$$C = 180\ \mu F$$

$$f = 50\ Hz$$