CONCEPT OF RESONANCE

In an electric circuit, reactive resistances in its various sections can be positive and negative and in some cases can be mutually compensated. When the reactive resistances in the circuit are fully compensated, its input resistance is active, the input voltage and current will coincide in phase, and the electric circuit will neither consume nor generate reactive power. The mode of operation of an electric circuit, if its input resistance is active in the presence of reactive resistances in it, is called resonant. The condition of the resonant mode is that the input reactive resistance or reactive conductivity is equal to zero
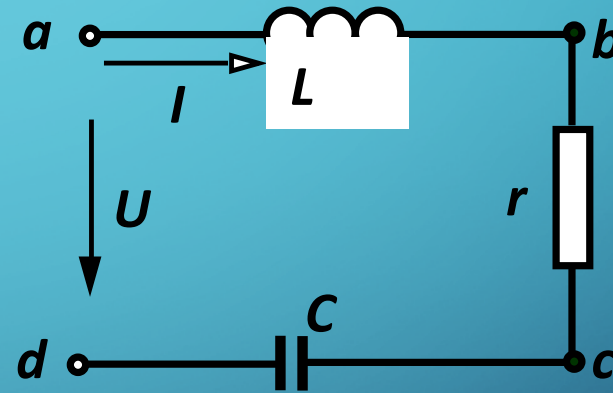
$$x_{вх} = 0$$

$$b_{вх} = 0$$

Frequencies at which resonance occurs in the circuit are called resonant frequencies

RESONANCE IN A SERIES CIRCUIT (VOLTAGE RESONANCE)

Consider a circuit (drawing) consisting of serially connected elements r, L, C and write down the complex input resistance of the circuit

$$\underline{Z}_{\text{вх}} = r + j\left(\omega L - \frac{1}{\omega C}\right)$$

According to the condition, resonance in this circuit is possible if the capacitive resistance is equal to the inductive resistance

$$X_{BX} = \omega L - \frac{1}{\omega C} = 0$$

It can be seen from the previous equation that the resonant mode can be achieved by changing either the capacitance, or the inductance, or the frequency. At constant L and C parameters, the resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

Effective value of the current in the circuit

$$I = \frac{U}{\sqrt{r^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}.$$

If the amplitude of the applied voltage is constant, then the current in the circuit at the moment of resonance acquires a maximum value

$$I_0 = \frac{U}{r}$$

and, as already noted, coincides in phase with the applied voltage.

The voltages on the reactive elements $\omega = \omega_0$ are equal to each other

$$U_{0L} = I_0 \omega_0 L = U \frac{\omega_0 L}{r}$$

$$U_{0C} = I_0 \frac{1}{\omega_0 C} = U \frac{1}{\omega_0 C r}.$$

If

$$\omega_0 L = \frac{1}{\omega_0 C} \gg r$$

then the voltages on the reactive elements significantly exceed the input voltage, therefore resonance in the series circuit is called voltage resonance.

The value of reactive resistance at the moment of resonance

$$\omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} = \rho$$

is called characteristic resistance of the loop.

An important parameter of the resonant circuit is its Q factor, which is the ratio of one of the reactive resistances at resonance to the active resistance of the circuit

$$Q = \frac{\omega_0 L}{r} = \frac{1}{\omega_0 C r} = \frac{\rho}{r} = \frac{U_{0L}}{U} = \frac{U_{0C}}{U}$$

Q-factor shows how many times the voltage on each of the reactive elements in the resonant mode exceeds the applied input voltage. Q-factor of circuits used in radio engineering is usually 50-300.
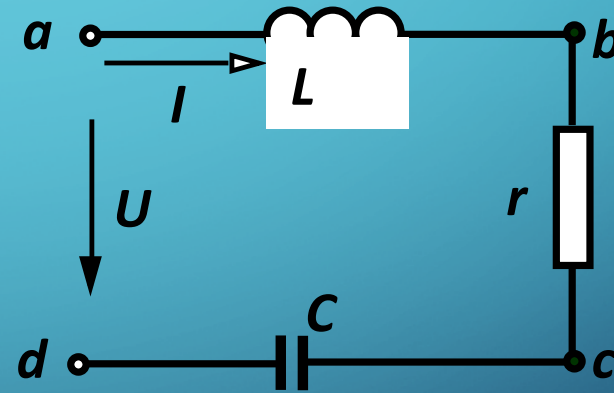
EXAMPLE
(INDIVIDUAL WORK)

Calculate the frequency for the cyclic frequency of resonance. Find the Q-factor of the circuit. Calculate the characteristic resistance of the circuit.
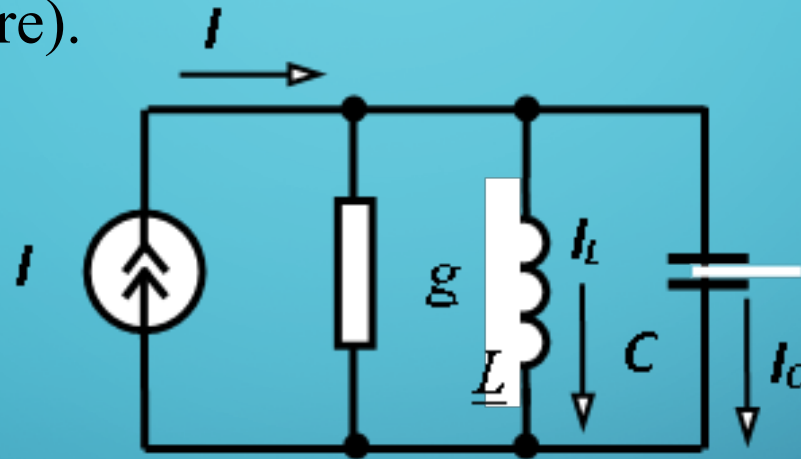
$L = 110 \; mH$

$C = 200 \; \mu F$

$r = 10 \; Ohm$

RESONANCE IN A PARALLEL CIRCUIT (RESONANCE OF CURRENTS)

Let an ideal current source be connected to parallel-connected elements g, L, C in a parallel ideal circuit (figure).

Complex input circuit conductivity

$$\underline{Y}_{вх} = g - j\left(\frac{1}{\omega L} - \omega C\right)$$

whence the resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

If the circuit is powered by a voltage source, then in the resonance mode the input current acquires a minimum value, $I_0 = Ug$ and the currents in circuits with reactive elements can exceed the input current many times, therefore such resonance is called current resonance.

Q-factor in this case is defined as the ratio of the current in any reactive element in the resonant mode to the current
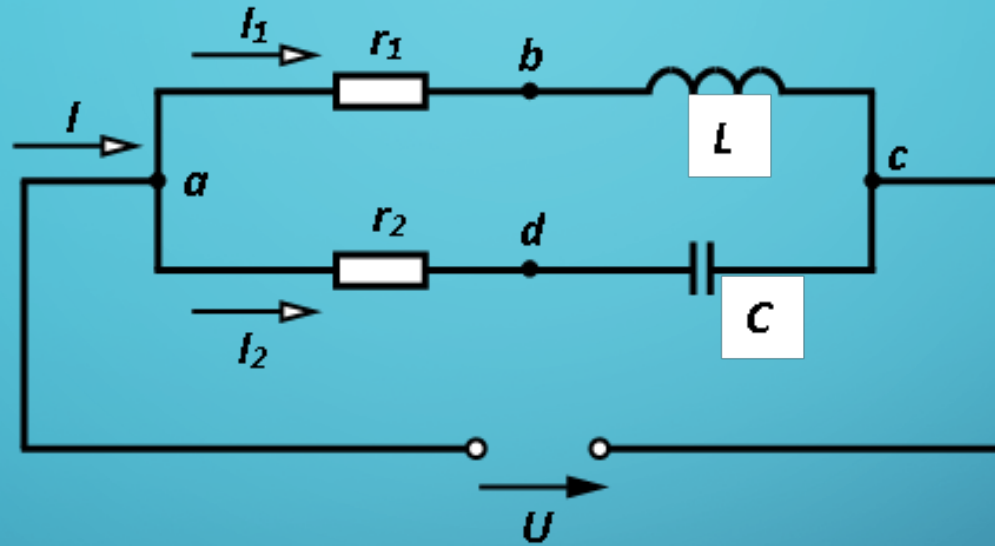
$$Q = \frac{I_{L0}}{I_0} = \frac{U}{\omega_0 L U g} = \frac{1}{\omega_0 L g} = \frac{\sqrt{\dfrac{C}{L}}}{g}$$

The value $\sqrt{C/L}$ has the dimension of conductivity and is called characteristic conductivity γ

From here

$$Q = \frac{\gamma}{g}$$

Consider the more general case when there are two parallel branches in the circuit, which contain inductance and capacitance with losses (figure).

The condition of resonance is that the reactive component of the input resistance or input conductivity is equal to zero $b_1 + b_2 = 0$

Because

$$\underline{Y}_1 = \frac{1}{r_1 + jx_1} = \frac{r_1}{r_1^2 + x_1^2} - j\frac{x_1}{r_1^2 + x_1^2}$$

$$\underline{Y}_2 = \frac{1}{r_2 + jx_2} = \frac{r_2}{r_2^2 + x_2^2} - j\frac{x_2}{r_2^2 + x_2^2}$$

where

$$x_1 = \omega L, \ \ x_2 = -\frac{1}{\omega C}$$

Then the resonance condition can be written as:

$$\frac{\frac{1}{\omega_0 C}}{r_2^2 + \left(\frac{1}{\omega_0 C}\right)^2} - \frac{\omega_0 L}{r_1^2 + (\omega_0 L)^2} = 0$$

After simple transformations, we determine the resonant frequency:

$$\omega_p = \frac{1}{\sqrt{LC}} \sqrt{\frac{r_1^2 C - L}{r_2^2 C - L}}$$

Taking into account that

$$\frac{L}{C} = \rho^2$$

let's write

$$\omega_p = \frac{1}{\sqrt{LC}} \sqrt{\frac{r_1^2 - \rho^2}{r_2^2 - \rho^2}}$$

Depending on the ratio between r1, r2 and ρ, there may be different cases. At $r_1 > \rho, r_2 < \rho$ or $r_1 < \rho, r_2 > \rho$ there will be no resonance at any frequency.

When $r_1 \ll \rho, r_2 \ll \rho$ the resonant frequency is the same as for an ideal circuit

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

When $r_1 = r_2 = \rho$ the value of the resonance frequency will be undefined. Resonance takes place at any frequency (indifferent resonance)

THANK FOR YOUR ATTENTION!