VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of General Engineering Sciences and Labour Safety

DECOMPOSITION OF SIGNALS INTO A FOURIER SERIES

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A SUM OF SINUSOIDS

OUR BUILDING BLOCK:

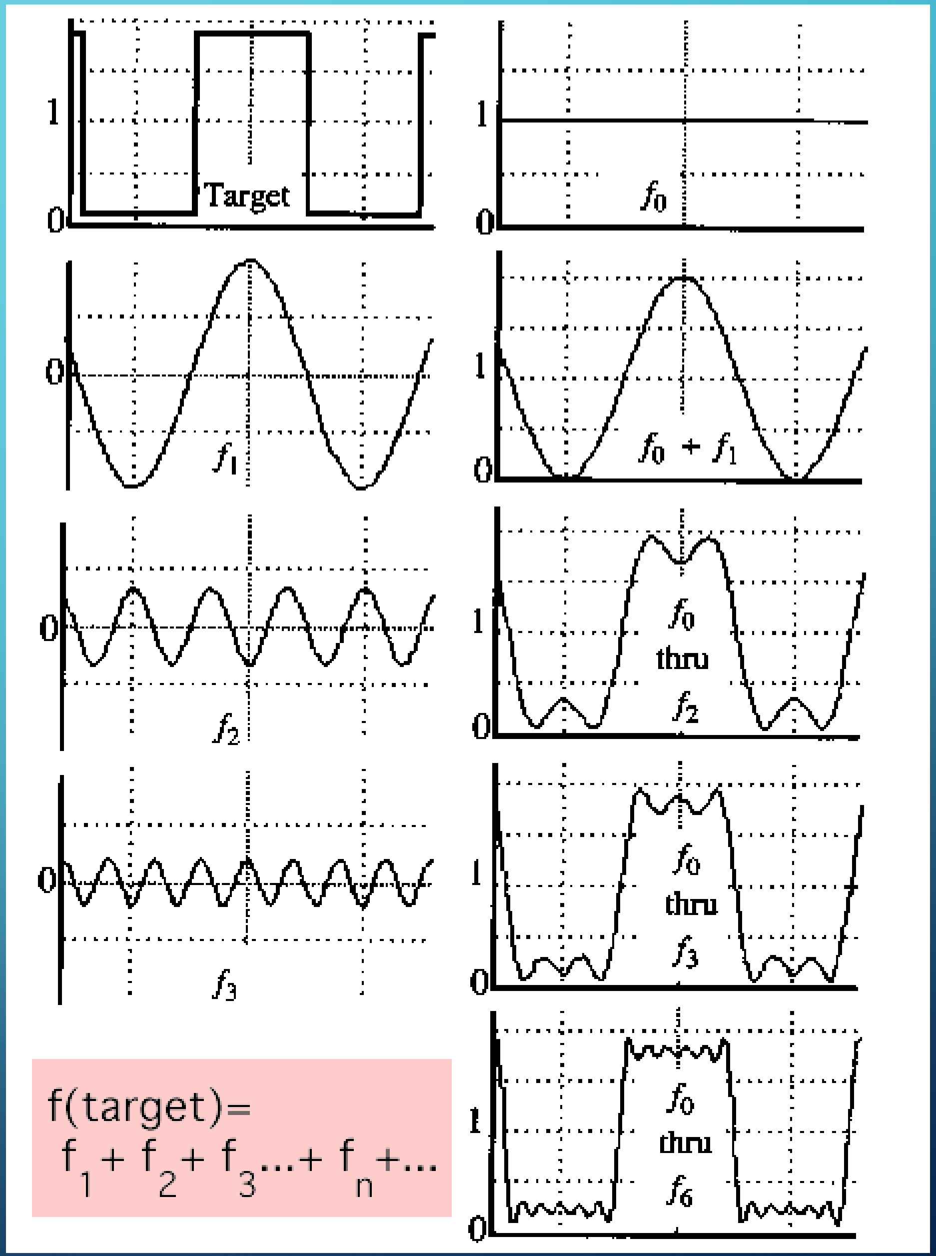
$$A\sin(\omega x + \phi)$$

ADD ENOUGH OF THEM TO GET ANY SIGNAL $F(X)$ YOU WANT!

HOW MANY DEGREES OF FREEDOM?

WHAT DOES EACH CONTROL?

WHICH ONE ENCODES THE COARSE VS. FINE STRUCTURE OF THE SIGNAL?



FOURIER TRANSFORM

WE WANT TO UNDERSTAND THE FREQUENCY ω OF OUR SIGNAL. SO, LET'S REPARAMETRIZE THE SIGNAL BY ω INSTEAD OF X:

f(x) → Fourier Transform → ***F(ω)***

- For every ω from 0 to inf, ***F(ω)*** holds the amplitude *A* and phase ϕ of the corresponding sine $A\sin(\omega x + \phi)$
  - How can *F* hold both? Complex number trick!

$$F(\omega) = R(\omega) + iI(\omega)$$

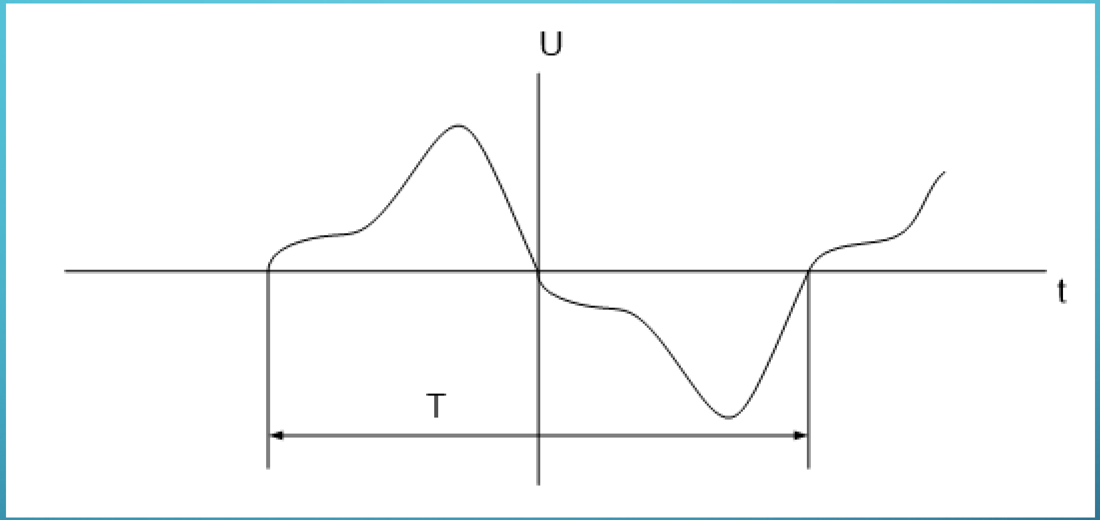
$$A = \pm\sqrt{R(\omega)^2 + I(\omega)^2} \qquad \phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$$

F(ω) → Inverse Fourier Transform → ***f(x)***

DECOMPOSITION OF NON-SINUSOIDAL CURRENTS AND VOLTAGES IN THE FOURIER SERIES

PERIODIC CURRENTS AND VOLTAGES THAT VARY ACCORDING TO LAWS OTHER THAN SINUSOIDAL (HARMONIC) ARE CALLED NON-SINUSOIDAL.

The presence of energy sources in electric circuits, the voltage of which, although periodic, but different from the harmonic, does not allow for the calculation of such circuits to use directly the method of complex amplitudes.



It is known that any periodic function f (x) that satisfies Dirichlet conditions, if the period of the function can be divided into a finite number of intervals, in each of which f (x) is continuous and monotonic, and at any breaking point f (x) there are f (x + 0) and f (x-0), can be represented by an infinite harmonic series:

$$B_0 + A_1 \sin x + B_1 \cos x + A_2 \sin 2x + B_2 \cos 2x + \ldots$$

FOURIER SERIES

$$f(x) = B_0 + \sum_{k=1}^{\infty} (A_k \sin kx + B_k \cos kx),$$

the Fourier series coefficients are determined by the following expressions

$$B_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)dx,$$

$$B_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos kxdx,$$

$$A_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin kxdx.$$

This expression can be simplified as follows:

$$f(x) = B_0 + \sum_{k=1}^{\infty} C_k \sin(kx + \beta\kappa)$$

where $C_K = \sqrt{A_K{}^2 + B_K{}^2}$ and $\beta_k = arktg \frac{B_k}{A_k}$

SIMPLIFICATION WHEN DECOMPOSING THE SIGNAL INTO A FOURIER SERIES

It should be noted that in cases of symmetry, some components of the Fourier series decomposition may be absent. If the function is odd $f(x) = -f(-x)$, then the decomposition will be no constant and component cosine components:

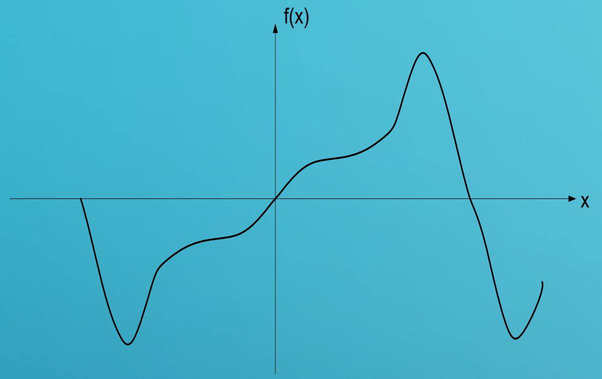


Рис. 6.4

$$f(x) = \sum_{k=1}^{\infty} A_k \sin kx.$$

If the function is even $f(x) = f(-x)$, then the decomposition will be no constant and component cosine components:

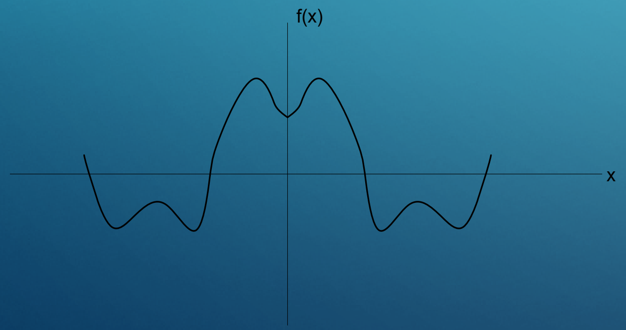


Рис. 6.3

$$f(x) = B_0 + \sum_{k=1}^{\infty} B_k \cos kx.$$

If the function is symmetric about the abscissa axis with an offset of half a period, then in the Fourier series distribution there will also be no components multiples of two

Write a Fourier series for voltage, depicting the expression in parentheses in the form of one sine wave:

$$U(t) = U_0 + U_{m1}\sin(\omega t + \beta_1) + U_{m2}\sin(2\omega t + \beta_2) + U_{m3}\sin(3\omega t + \beta_3) + ...$$

or

$$u(t) = U_0 + \sum_{k=1}^{\infty} U_{mk}\sin(k\omega t + \beta_k).$$

Each component in this expression is called a harmonic with a number multiple of the frequency of this component with the frequency of non-sinusoidal noise (voltage).Detailed and rigorous evidence of the written provisions can be found in mathematics textbooks. preliminary information about the properties of the functions can significantly reduce the calculations.

EXAMPLE. Determine the coefficients of the Fourier series of the periodic function, which is shown in Fig.6.6.

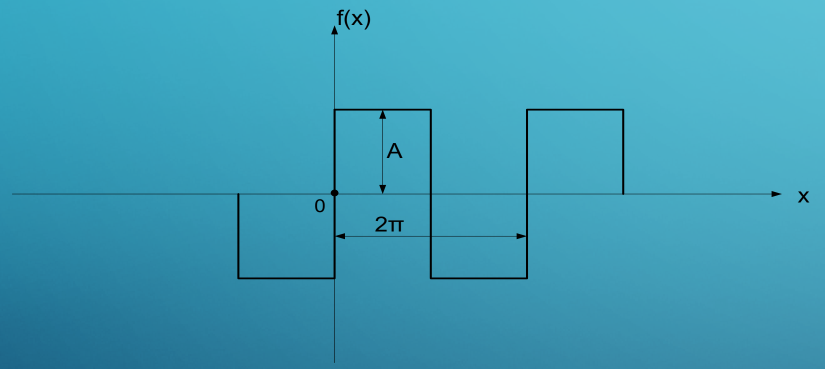


Рис. 6.6

$$f(x) = \begin{cases} A, & 0 < x < \pi \\ -A, & \pi < x < 2\pi \end{cases}$$

SOLUTION: The function under consideration is symmetric with respect to the origin and with respect to the abscissa, so only sinusoidal components of odd harmonics will be present in the curve schedule.

$$f(x) = A_1\sin x + A_3\sin 3x + A_5\sin 5x + ...$$

COEFFICIENTS

- Coefficient A_k are defined according to the expression:

$$A_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx$$

- Considering $f(x) = \begin{cases} A, & 0 < x < \pi \\ -A, & \pi < x < 2\pi \end{cases}$ we have $A_k = \frac{1}{\pi}\left[\int_0^{\pi} A\sin kx dx - \int_0^{2\pi} A\sin kx dx\right]$

- or $A_k = \frac{4A}{\pi \cdot k} \quad (k = 1,3,5....)$

- As follows $f(x) = \frac{4A}{\pi}(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + ...)$

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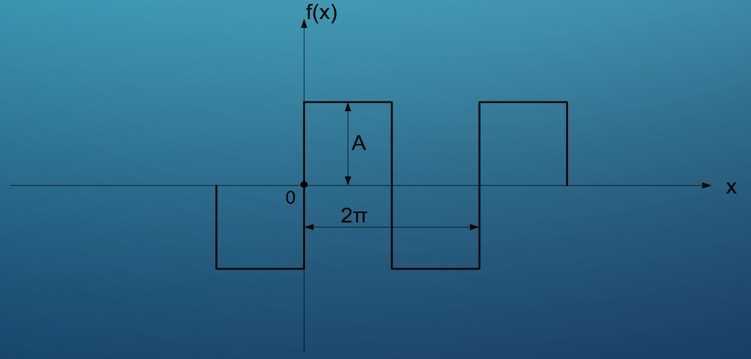


Рис. 6.6

EFFECTIVE (RMS) VALUE OF NONSINUSOIDAL CURRENTS AND VOLTAGES

- RNS current

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt}.$$

- *Determine the effective value of non-sinusoidal current*

$$i = I_0 + I_{m1}\sin(\omega t + \alpha_1) + I_2\sin(2\omega t + \alpha_2) + \ldots \quad .$$

- *For each of its components, after summing the expression to the square we have:*

$$\int_0^T I_0^2 dt = I_0^2 T, \qquad \int_0^T 2I_0 I_{mk}\sin(k\omega t + \alpha_k)dt = 0, \qquad \int_0^T I_{mk}^2 \sin^2(k\omega t + \alpha_k) = \int_0^T \frac{I_{mk}^2}{2}[1 - \cos 2(k\omega t + \alpha_k)]dt = \frac{I_{mk}^2}{2}T,$$

- *For* $\kappa \neq s$ $\int_0^T I_{mk}\sin(k\omega t + \alpha_k) I_{ms}\sin(s\omega t + \alpha_3)dt = \int_0 \frac{I_{mk}I_{ms}}{2}\{\cos[(k-s)\omega t + \alpha_k - \alpha_s] - \cos[(k+s)\omega t + (\alpha_k + \alpha_s)]\}dt = 0.$
- *So as a result we get:*

$$I = \sqrt{I_0^2 + \frac{I_{m1}^2}{2} + \frac{I_{m2}^2}{2} + \frac{I_{m3}^2}{2} + \ldots}$$

- *or* $I = \sqrt{I_0^2 + I_1^2 + I_2^2 + I_3^2 + \ldots}$ *similarly* $U = \sqrt{U_0^2 + U_1^2 + U_2^2 + U_3^2 + \ldots}$
- *The effective value of non-sinusoidal current (voltage) is equal to the square root of the sum of squares of the effective values of individual harmonics*

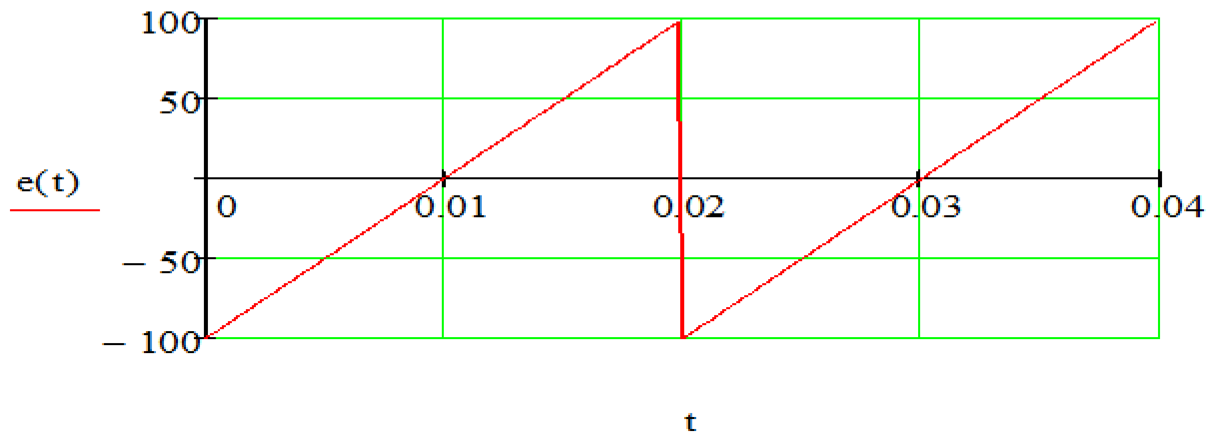
EXAMPLE

$b := -100 \qquad A := 100 \qquad k := \frac{2 \cdot A}{T} \qquad f1(t) := k \cdot t + b$

$f2(t) := k \cdot t + 3 \cdot b$

$$e(t) := \begin{cases} f1(t) & \text{if } 0 \le t < T \\ f2(t) & \text{if } T \le t < 2 \cdot T \end{cases}$$

$$t := 0, \frac{T}{100} .. 2 \cdot T$$



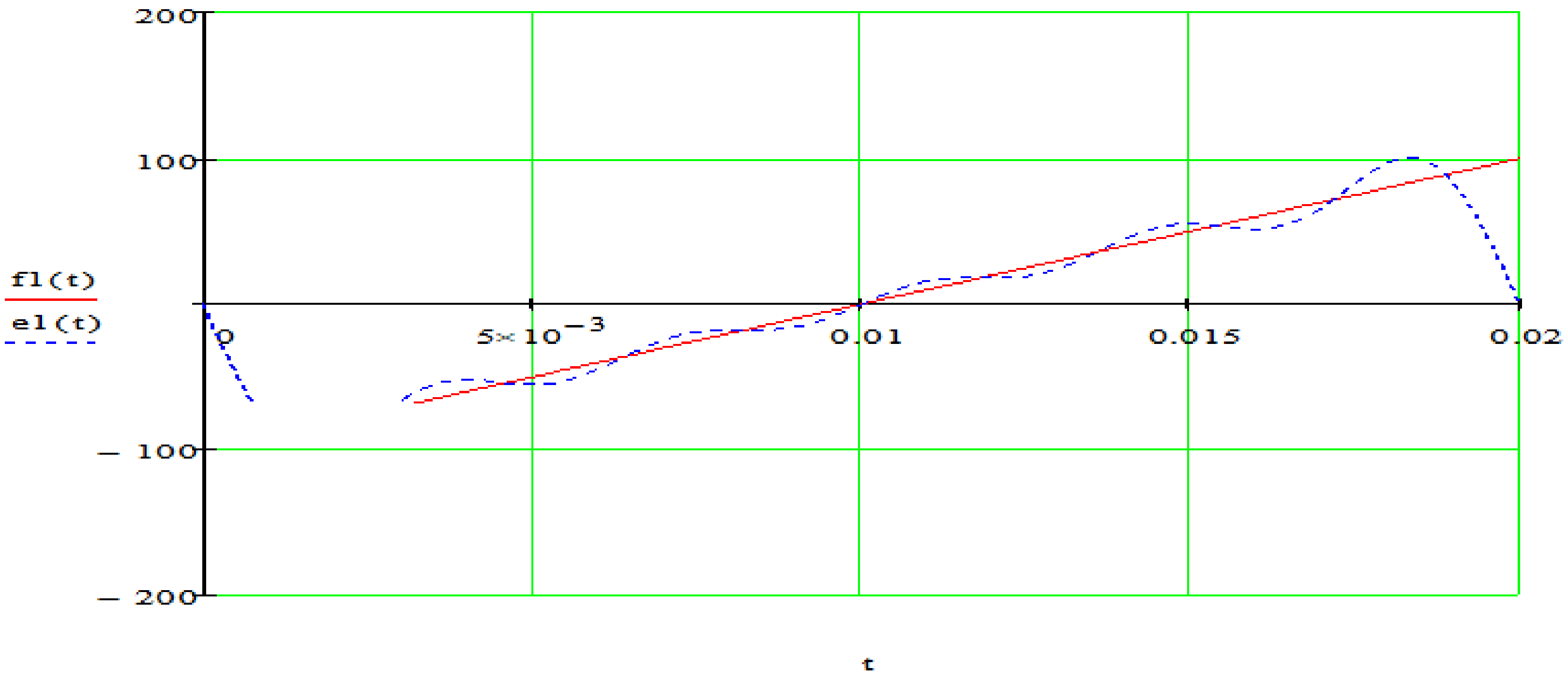
EXAMPLE OF DECOMPOSITION INTO A FOURIER SERIES

Harmonic amplitude

$$Um_k := \frac{2 \cdot \int_0^T f1(t) \cdot \sin(k \cdot w \cdot t)\, dt}{T}$$

$$e1(t) := \sum_{k=1}^{n} \left(Um_k \cdot \sin(k \cdot w \cdot t)\right)$$

$$t := 0, \frac{T}{100} .. T$$

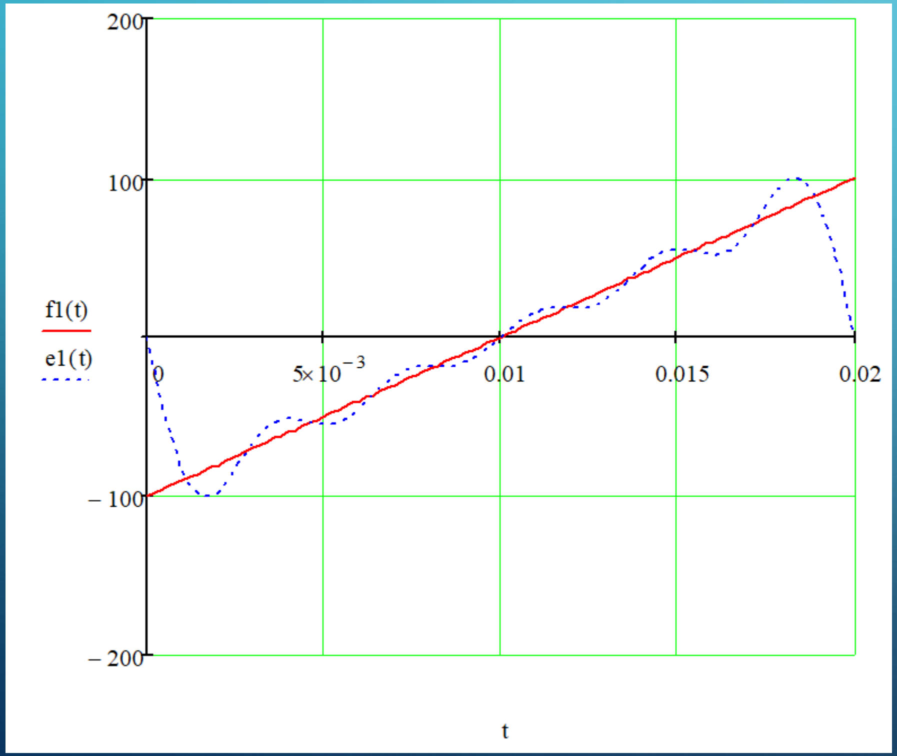


$$n := 5$$

$$k := 1 .. n$$

$$Um_k := \frac{2 \cdot \int_0^T f1(t) \cdot \sin(k \cdot w \cdot t)\, dt}{T}$$

$$e1(t) := \sum_{k=1}^{n} \left(Um_k \cdot \sin(k \cdot w \cdot t) \right)$$



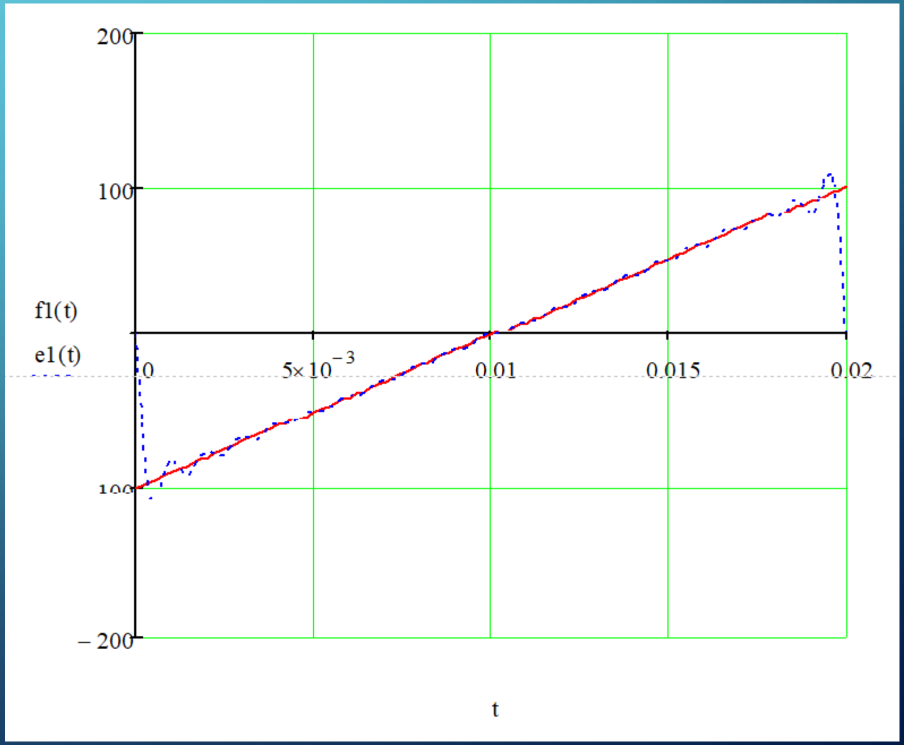
$$n := 20$$

$$k := 1 .. n$$

$$Um_k := \frac{2 \cdot \int_0^T f1(t) \cdot \sin(k \cdot w \cdot t)\, dt}{T}$$

$$e1(t) := \sum_{k=1}^{n} \left(Um_k \cdot \sin(k \cdot w \cdot t) \right)$$

$$t := 0, \frac{T}{100} .. T$$



EXAMPLE OF CALCULATION (individual work)

Calculate the amplitudes of the zero and first harmonics