VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of Electric Power Engineering, Electrical Engineering and Electromechanics

METHOD OF NODAL POTENTIALS

by Associate Professor V. Hraniak

THE ESSENCE AND MAIN APPROACHES

When an electric circuit consists of a large number of loops with a small number of nodes, it is advisable to perform its calculation and analysis by the method of nodal potentials (or the method of nodal voltages).

If the number of nodes in the scheme is n, then the number of equations needed to calculate such a circle is equal to (n – 1). The unknown quantities in these equations are the so-called nodal potentials. In accordance with this method, the potential in one of the circuit nodes is assumed to be zero. Other nodes of the circuit will have potentials (nodal voltages U_1, U_2, ..., U_{n-1}) relative to the node with zero potential

The current in each branch of the circuit is determined by the voltages applied to the nodes of the branch, their EMF and resistances of the branches.

Next, using the expressions for currents, make equations according to Kirchhoff's second rule for each node of the circuit, excluding the node with zero voltage. The set of such equations forms a system of equations relative to the unknown nodal voltages.

When compiling the equation for any i-th node, you can use the ready-made universal formula:

$$\phi_{\mathrm{i}} \sum_{\substack{j=1 \\ j=i}}^{n} \frac{1}{R_{\mathrm{ij}}} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \left(\varphi_{\mathrm{j}} \frac{1}{R_{\mathrm{ij}}} \right) = \sum_{\substack{j=1 \\ j=i}}^{n} \left(E_{\mathrm{ij}} \frac{1}{R_{\mathrm{ij}}} \right)$$

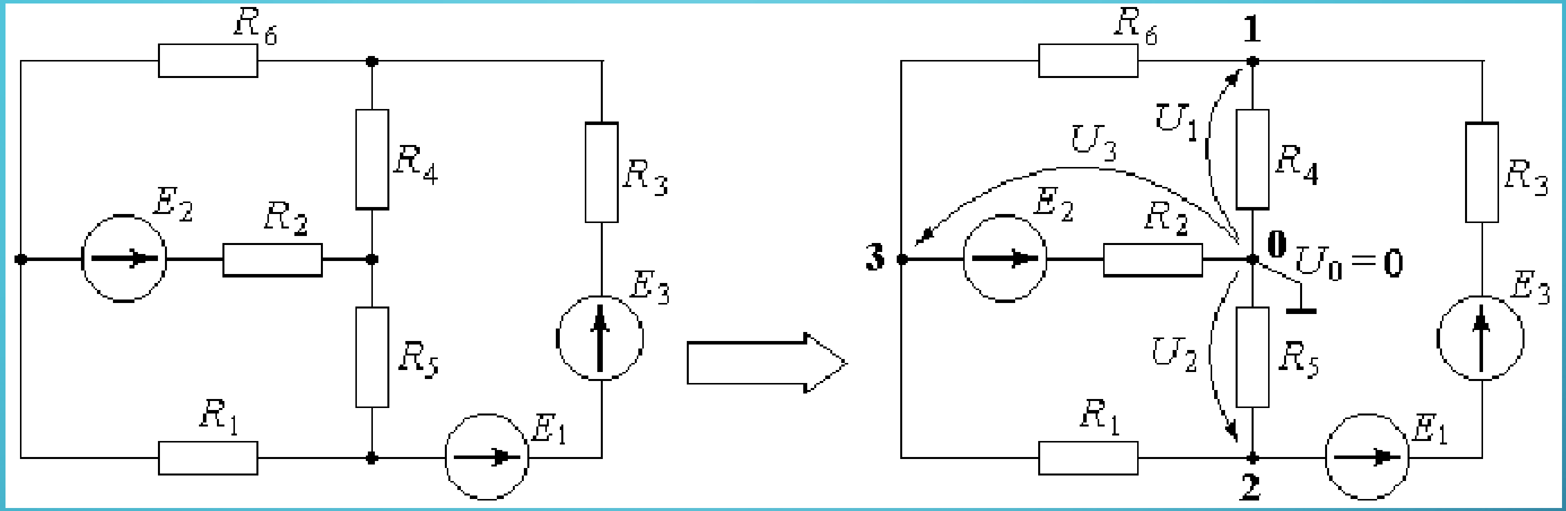
according to which:

the result of multiplication nodal potential in the i-th node by the sum of the conductances of the branches between the i-th and each of the nodes adjacent to the i-th node, minus the sum of the products of the nodal potentials in each node adjacent to the i-th node by the conductance of the branch between this node and the i-th, is equal to the sum of the products of the EMF in the branch between i–th and each node adjacent to i–th (if it is present in this branch) by the conductivity of this branch.

The components Eij are taken with a "+" sign if the EMF is directed to the i-th node and with a "–" sign if it is directed from the i-th node.

Having solved the system with respect to φ_i, it is possible to determine the currents in the branches.

Consider the calculation of an electric circuit using this method using the example of the circuit shown in the figure



We arbitrarily number the nodes of the scheme, starting from zero. We take the potential at node No. 0 as zero. Using the given formula, we make equations for other nodes:

Node number 1: $$\varphi_1\left(\frac{1}{R_6}+\frac{1}{R_4}+\frac{1}{R_3}\right)-\varphi_2\frac{1}{R_3}-\varphi_3\frac{1}{R_6}=(E_1+E_3)\frac{1}{R_3};$$

Node number 2: $$-\varphi_1\frac{1}{R_3}+\varphi_2\left(\frac{1}{R_1}+\frac{1}{R_5}+\frac{1}{R_3}\right)-\varphi_3\frac{1}{R_1}=(-E_1-E_3)\frac{1}{R_3};$$

Node number 3: $$-\varphi_1\frac{1}{R_6}-\varphi_2\frac{1}{R_1}+\varphi_3\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_6}\right)=-E_2\frac{1}{R_2}.$$

Given the following ratios:

$$Y = \frac{1}{R}, \qquad I = E\frac{1}{R},$$

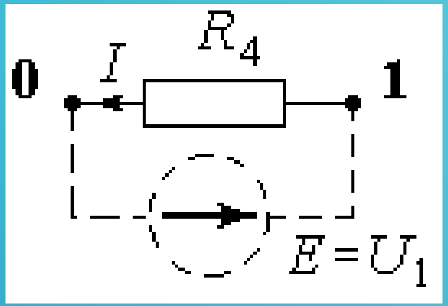
the initial formula can be written in the form of the following generalized system of equations:

$$\begin{cases} \phi_1 Y_{11} - \phi_2 Y_{12} - \ldots - \phi_n Y_{1n} = I_{11}, \\ -\phi_1 Y_{21} + \phi_2 Y_{22} - \ldots - \phi_n Y_{2n} = I_{22}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -\phi_1 Y_{n1} - \phi_2 Y_{n2} - \ldots + \phi_n Y_{nn} = I_{nn}, \end{cases}$$

where Y_{ii} is the own conductance of the i-th node, defined as the sum of the conductances of the branches connected to this node; Y_{ij} is the joint conductivity of the i-th and j-th nodes, defined as the sum of the conductances of the branches simultaneously connected to the i-th and j-th nodes; Iii is the own current of the i-th node, defined as the sum of the products of the EMF on the conductivity of the active loops connected to the i-th node (in the case when the EMF is directed to the i-th node, its direction is considered positive, in the other case - negative).

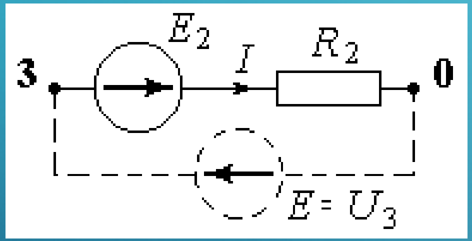
After solving the system of equations relative to the unknowns φ_1, φ_2, φ_3, that is, determining their values, we calculate the currents in the circuits.

The branch with nodes 0-1.



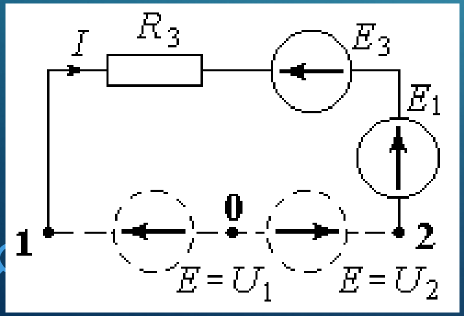
The action of the voltage $U_1=\varphi_1-\varphi_4$ applied to the nodes is equivalent to the action of the included emf $E = U_1$. For such an artificially formed contour, the equation $IR_4 = U_1$ is formed according to Kirchhoff's second rule. From which: $I = U_1/R_4$.

The branch with nodes 0-3.



$$U_3 = \varphi_3 - \varphi 4 \quad IR_2 = E_2 + U_3 \quad \Rightarrow \quad I = (E_2 + U_3)/R_5.$$

The branch with nodes 1-2.



$$IR_3 = -E_3 - E_1 - U_2 + U_1 \quad \Rightarrow \quad I = (-E_3 - E_1 - U_2 + U_1)/R_3.$$

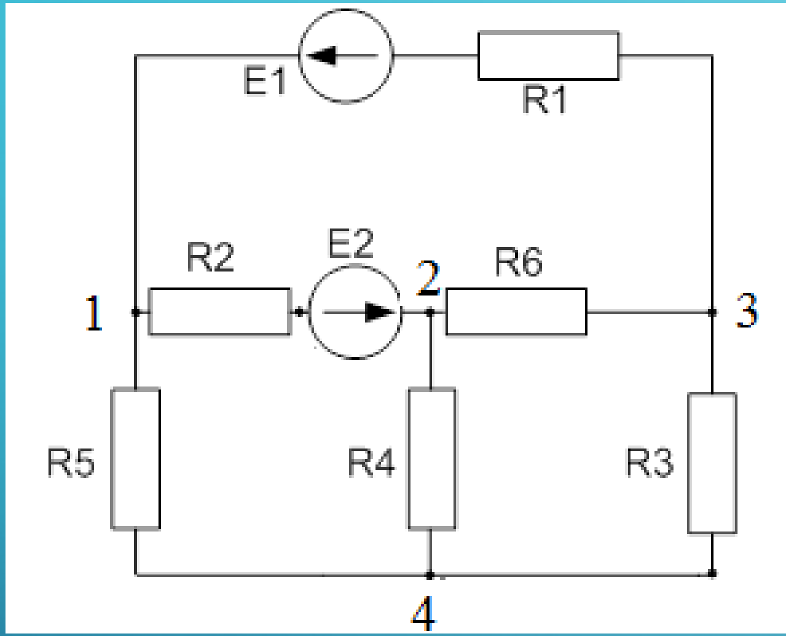
Similarly for other branches.

ALGORITHM FOR SOLVING AN ELECTRIC CIRCUIT BY THE METHOD OF NODAL POTENTIALS

1. Take the potential of one of the nodes as zero (ground it).
2. Compile equations using the method of nodal potentials. The number of equations is equal to the number of remaining nodes.
3. Using any calculation method, solve the system of equations and determine the potentials of the nodes.
4. Based on the found nodal potentials, determine the currents in the branches of the circuit.

EXAMPLE

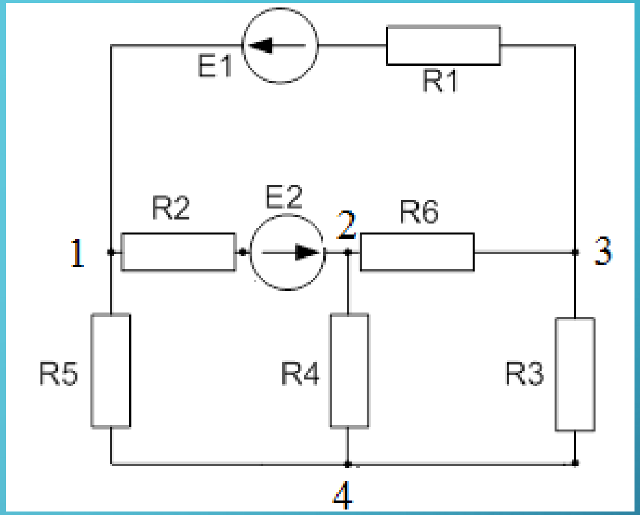
For the circle depicted in the figure, make a system of equations using the method of nodal potentials



Ground node 4 and write down the system of equations using the nodal potential method in general form:

$$\begin{cases} \phi_1 Y_{11} - \phi_2 Y_{12} - \phi_3 Y_{13} = I_{11}, \\ -\phi_1 Y_{21} + \phi_2 Y_{22} - \phi_2 Y_{23} = I_{22}, \\ -\phi_1 Y_{31} - \phi_2 Y_{32} + \phi_3 Y_{33} = I_{33}. \end{cases}$$

Let's determine the own and joint conductances of nodes and node currents.



$$Y_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5},$$

$$Y_{22} = \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_6},$$

$$Y_{33} = \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_6},$$

$$Y_{12} = Y_{21} = \frac{1}{R_2},$$

$$Y_{13} = Y_{31} = \frac{1}{R_1},$$

$$Y_{23} = Y_{32} = \frac{1}{R_6},$$

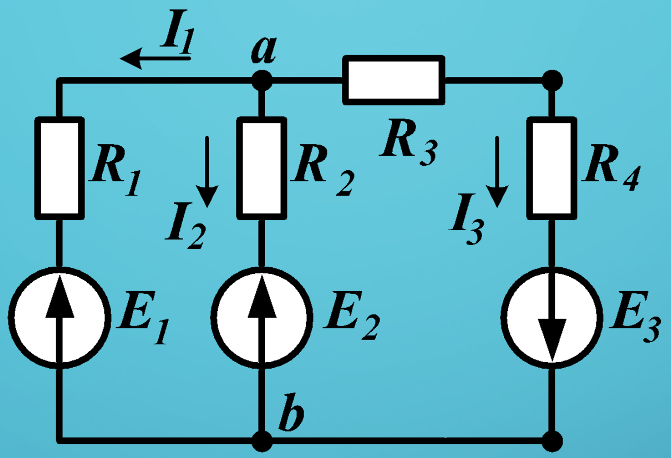
$$I_{11} = E_1 \frac{1}{R_1} - E_2 \frac{1}{R_2},$$

$$I_{22} = E_2 \frac{1}{R_2},$$

$$I_{33} = -E_1 \frac{1}{R_i}.$$

EXAMPLE OF CALCULATION (individual work)

For the circle depicted in the figure, make a system of equations using the method of nodal potentials

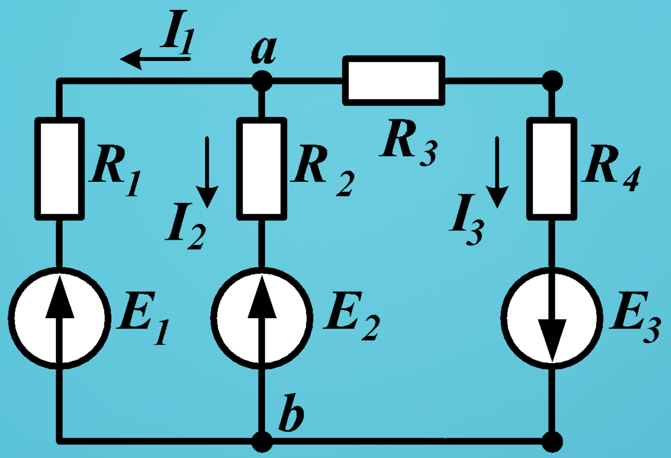


$$\phi_a = \phi_1 \qquad \phi_b = 0$$

$$\phi_1 Y_{11} = I_{11}$$

$$Y_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3 + R_4} \qquad I_{11} = \frac{E_1}{R_1} + \frac{E_2}{R_2} - \frac{E_3}{R_3 + R_4}$$

CALCULATION OF CURRENTS



$$I_1 = \frac{\phi_1 - E_1}{R_1}$$

$$I_2 = \frac{\phi_1 - E_2}{R_2}$$

$$I_3 = \frac{\phi_1 + E_3}{R_3 + R_4}$$

THANK FOR YOUR ATTENTION!