VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of Electric Power Engineering, Electrical Engineering and Electromechanics
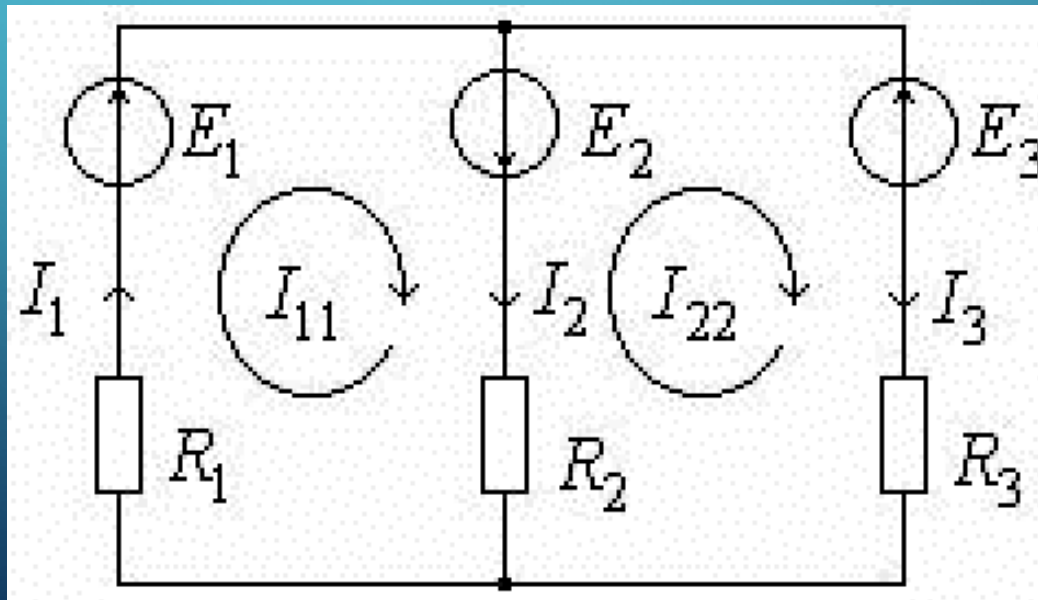

METHOD OF LOOP CURRENTS
by Associate Professor V. Hraniak

THE ESSENCE OF METHOD OF LOOP CURRENTS

The method is based on Kirchhoff's rules. When analyzing a circuit, it is assumed that each independent loop has its own loop current. Equations for loop currents are calculated according to Kirchhoff's second rule. The number of equations is equal to the number of independent loops, that is, the method of loop currents is more economical in computational work.

The figure shows a circle with two independent contours. I1, I2, I3 – currents in the circuit branches, I11, I22 – loop's currents.

According to Kirchhoff's second rule:

$$I_1 R_1 + I_2 R_2 = E_1 + E_2,$$
$$-I_2 R_2 + I_3 R_3 = -E_2 - E_3.$$

Express the currents in the branches in terms of loop currents:

$$I_{11} R_1 + (I_{11} - I_{22}) R_2 = E_1 + E_2,$$
$$-(I_{11} - I_{22}) R_2 + I_{22} R_3 = -E_2 - E_3,$$
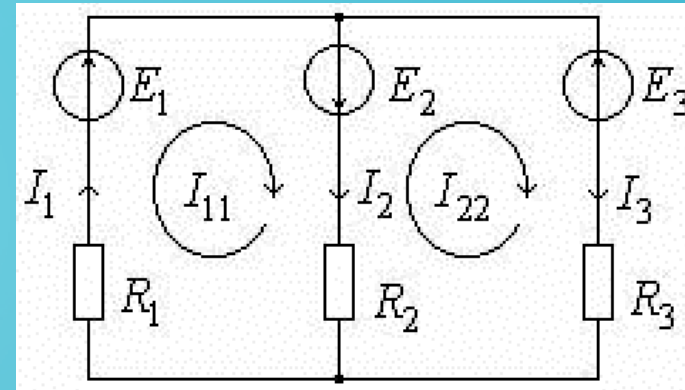
or

$$I_{11}(R_1 + R_2) - I_{22} R_2 = E_1 + E_2,$$
$$-I_{11} R_2 + I_{22}(R_2 + R_3) = -E_2 - E_3.$$

Obviously, the currents in the external circuits are equal to the loop currents :
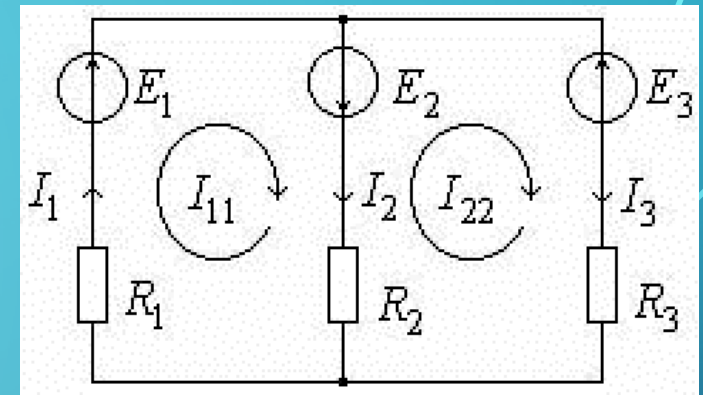
$$I_1 = I_{11}, \quad I_3 = I_{22}.$$

In the internal circuits, the currents are equal to the difference of the loop currents:

$$I_2 = I_{11} - I_{22}.$$

Let's enter the notation:

$$R_1 + R_2 = R_{11}$$ - own resistance of the first loop (the sum of the resistances of the first loop

$$R_2 + R_3 = R_{22}$$ - own resistance of the second loop (the sum of the resistances of the first second

$$-R_2 = R_{12} = R_{21}$$ - joint resistance of the first and second loop (the sum of the resistance of the resistors belonging to the first and second loop at the same time)

$$E_1 + E_2 = E_{11}$$ - own EMF of first loop

$$-E_2 - E_3 = E_{22}$$ - own EMF of second loop

WARNING!

If the direction of the EMF coincides with the selected direction of the loop current, enter the loop EMF with a "plus" sign.

For a given circuit, the system of equations according to the method of loop currents has the following general form:

$$\begin{cases} I_{11}R_{11} + I_{22}R_{12} = E_{11}, \\ I_{11}R_{21} + I_{22}R_{22} = E_{22}. \end{cases}$$

Similarly, the system of equations for a three-loop circuit is written:

$$\begin{cases} I_{11}R_{11} + I_{22}R_{12} + I_{33}R_{13} = E_{11}, \\ I_{11}R_{21} + I_{22}R_{22} + I_{33}R_{23} = E_{22}, \\ I_{11}R_{31} + I_{22}R_{32} + I_{33}R_{33} = E_{33}. \end{cases}$$

or in the form of a matrix:

$$[R] \cdot [I] = [E],$$

where

$$[R] = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}, \quad [I] = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \end{bmatrix}, \quad [E] = \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \end{bmatrix}.$$

ALGORITHM FOR SOLVING AN ELECTRIC CIRCUIT BY THE METHOD OF LOOP CURRENTS.

1. Determine the number of independent loops. Compile equations using according to method of loop currents. The number of equations composed by the method of loop currents should be equal to the number of independent loops.

2. Determine the own and joint resistances of loops and their EMF.

3. Use any calculation method to solve the system of equations and determine the loop currents.

4. Based on the found loop currents, determine the currents in the branches of the circuit.

EXAMPLE

For the circle shown in the figure, make a system of equations using the method of loop currents.
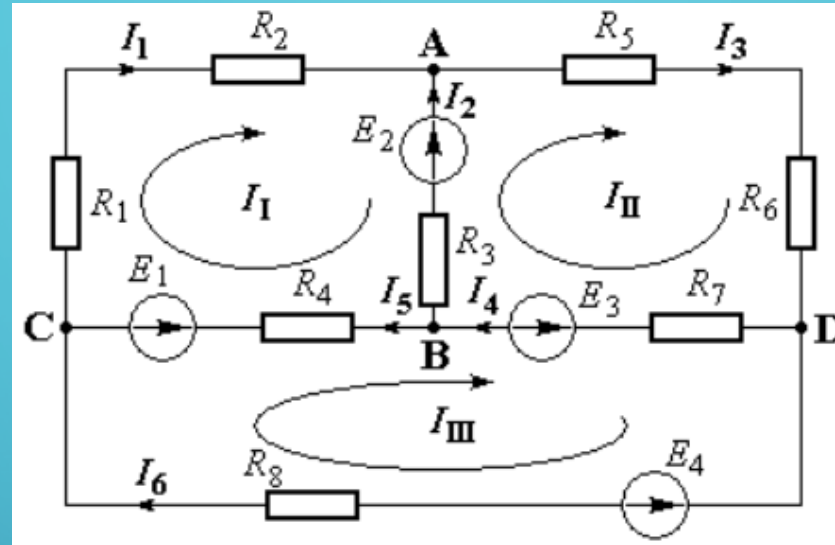
Let's select three independent loops in the circle and write the system of equations according to the method of loop currents in the general form:

$$\begin{cases} I_{11}R_{11} + I_{22}R_{12} + I_{33}R_{13} = E_{11}, \\ I_{11}R_{21} + I_{22}R_{22} + I_{33}R_{23} = E_{22}, \\ I_{11}R_{31} + I_{22}R_{32} + I_{33}R_{33} = E_{33}. \end{cases}$$

Let's determine the own and joint resistances of the circuits and the loop own EMF.

$$R_{11} = R_1 + R_2 + R_3 + R_4,$$

$$R_{22} = R_3 + R_5 + R_6 + R_7,$$

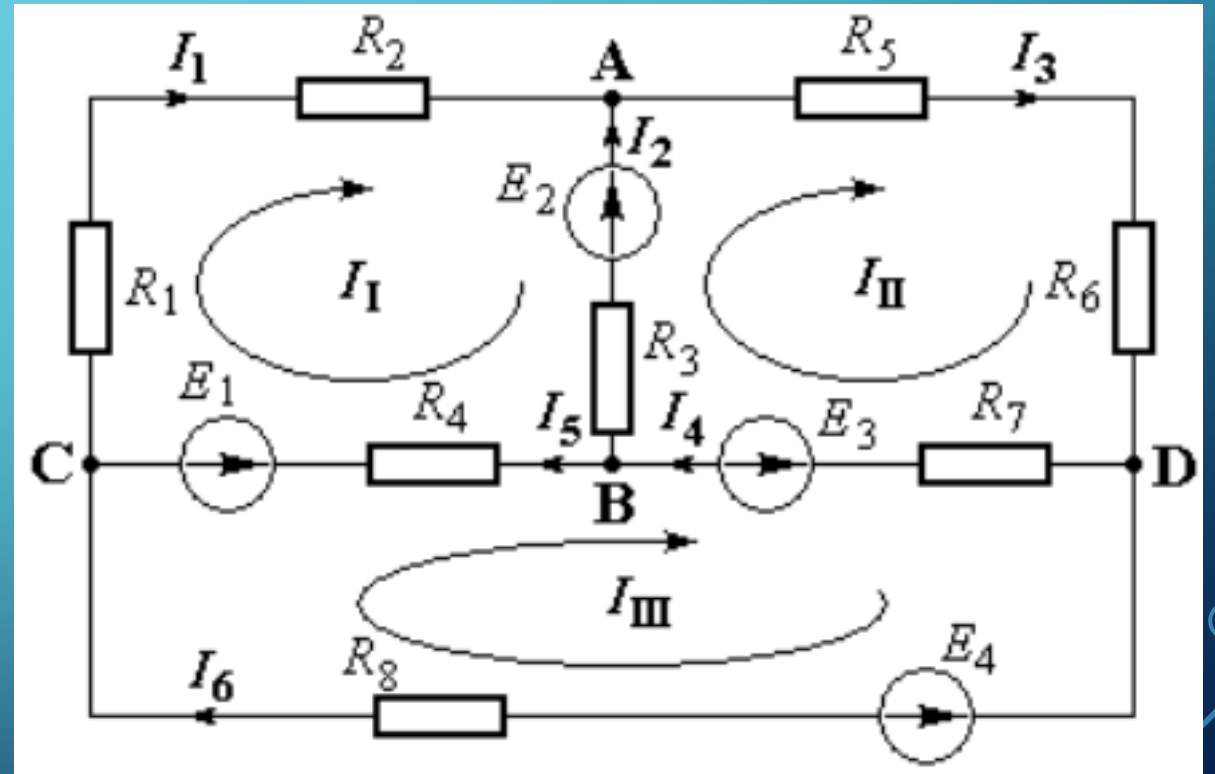$$R_{33} = R_4 + R_7 + R_8,$$

$$R_{12} = R_{21} = -R_3,$$

$$R_{13} = R_{31} = -R_4,$$

$$R_{23} = R_{32} = -R_7,$$

$$E_{11} = -E_1 - E_2,$$
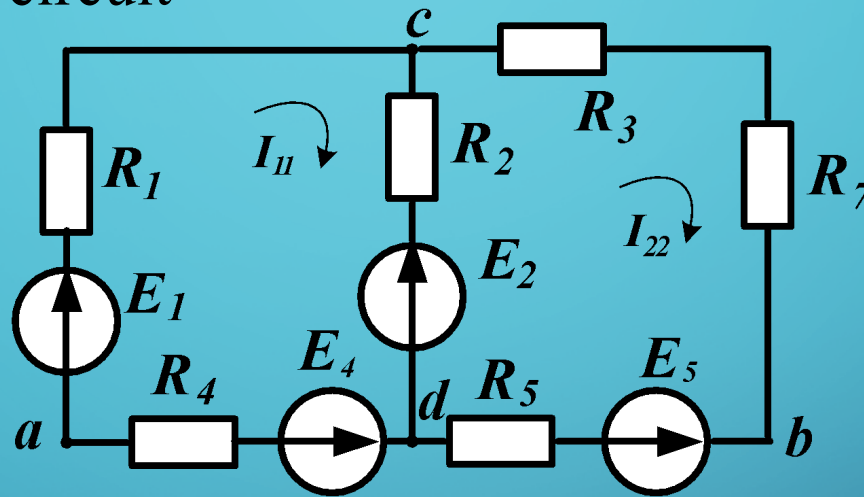
$$E_{22} = E_2 - E_3,$$

$$E_{33} = E_1 + E_3 - E_4.$$

EXAMPLE OF CALCULATION
(individual work)

Complete the system of equations by the method of loop currents and write its coefficients for the next circuit

$$I_{11}R_{11} + I_{22}R_{12} = E_{11}$$
$$I_{11}R_{21} + I_{22}R_{22} = E_{22}$$

$$R_{11} = R_1 + R_4 + R_2 \qquad\qquad E_{11} = E_1 - E_2 - E_4$$

$$R_{22} = R_3 + R_2 + R_5 + R_7 \qquad\qquad E_{22} = E_2 - E_5$$

PECULIARITIES OF THE LOOP CURRENT METHOD IF THERE ARE CURRENT SOURCES IN THE CIRCUIT

If the circuit has branche containing ideal current sources, it is convenient to choose the loops so that such a branch with current sources is included in only one loop. In this case, the loop current of this loop is already known - it is equal to the current of the current source, and the number of unknown loop currents decreases.

Example

Calculate all currents in the circuit in the figure by the method of loop currents with the following set parameters:

R1=26(Ohm), R2=10(Ohm),
R3=18(Ohm), R4=14(Ohm),
R5=20(Ohm),   R6=8(Ohm),
E2=20(V),   E7=24(V),
J=2(A).
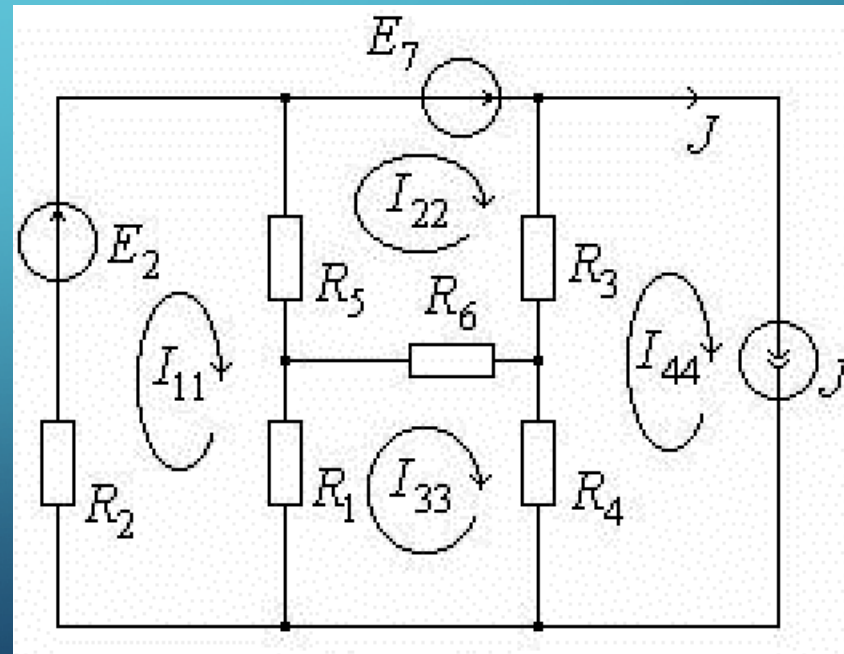
The circle has four independent loops. According to the method of loop currents, three equations need to be compiled, since the loop current $I_{44}=J$ is known. Let's write the system of equations according to the method of loop currents in general form:
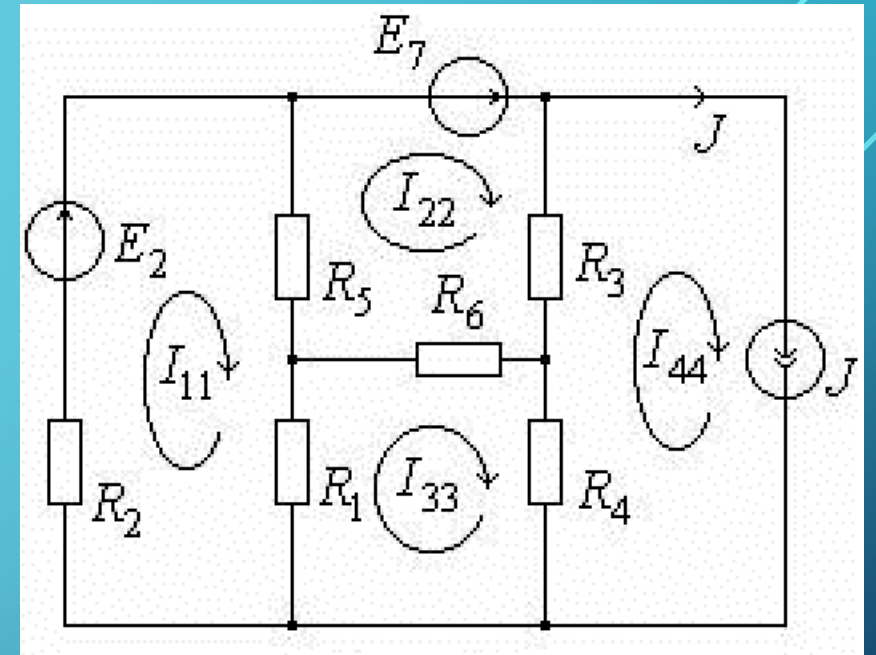
$$\begin{cases} I_{11}R_{11} + I_{22}R_{12} + I_{33}R_{13} + I_{44}R_{14} = E_{11}, \\ I_{11}R_{21} + I_{22}R_{22} + I_{33}R_{23} + I_{44}R_{24} = E_{22}, \\ I_{11}R_{31} + I_{22}R_{32} + I_{33}R_{33} + I_{44}R_{34} = E_{33}. \end{cases}$$

Own resistances:

$$R_{11} = R_1 + R_2 + R_5 = 26 + 10 + 20 = 56 \ (Ohm),$$

$$R_{22} = R_5 + R_3 + R_6 = 20 + 18 + 8 = 46 \ (Ohm),$$

$$R_{33} = R_1 + R_6 + R_4 = 8 + 14 + 26 = 48 \ (Ohm).$$
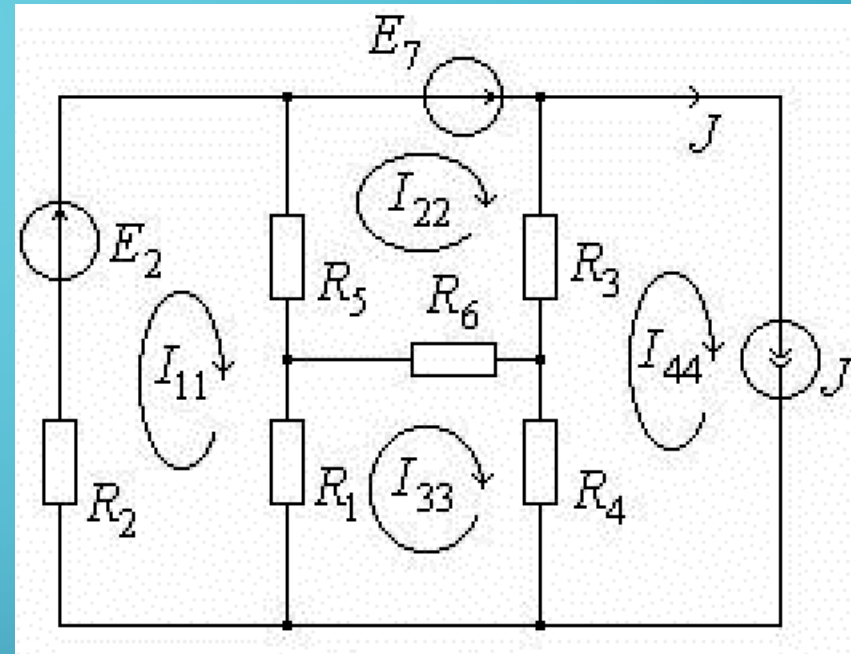
Joint resistances

$$R_{12} = R_{21} = -R_5 = -20 \ (Ohm),$$
$$R_{13} = R_{31} = -R_1 = -26 \ (Ohm),$$
$$R_{14} = R_{41} = 0 \ (Ohm),$$
$$R_{23} = R_{32} = -R_6 = -8 \ (Ohm),$$
$$R_{24} = R_{42} = -R_3 = -18 \ (Ohm),$$
$$R_{34} = R_{43} = -R_4 = -14 \ (Ом).$$

Own EMF:
$$E_{11} = E_2 = 20 \ (V),$$
$$E_{22} = E_7 = 24 \ (V),$$
$$E_{33} = 0 \ (V).$$

Let's substitute the values into the system of equations:

$$\begin{cases} 56I_{11} - 20I_{22} - 26I_{33} = 20, \\ -20I_{11} + 46I_{22} - 8I_{33} - 18 \cdot 2 = 24, \\ -26I_{11} - 8I_{22} + 48I_{33} - 14 \cdot 2 = 0. \end{cases}$$

or after simplification:

$$\begin{cases} 56I_{11} - 20I_{22} - 26I_{33} = 20, \\ -20I_{11} + 46I_{22} - 8I_{33} = 60, \\ -26I_{11} - 8I_{22} + 48I_{33} = 28. \end{cases}$$

Let's solve the system of equations using Kramer's method:

$$\Delta = \begin{bmatrix} 56 & -20 & -26 \\ -20 & 46 & -8 \\ -26 & -8 & 48 \end{bmatrix} = 61448,$$

$$\Delta_1 = \begin{bmatrix} 20 & -20 & -26 \\ 60 & 46 & -8 \\ 28 & -8 & 48 \end{bmatrix} = 150998,$$

$$\Delta_2 = \begin{bmatrix} 56 & 20 & -26 \\ -20 & 60 & -8 \\ -26 & 28 & 48 \end{bmatrix} = 171184,$$

$$\Delta_3 = \begin{bmatrix} 56 & -20 & 20 \\ -20 & 46 & 60 \\ -26 & -8 & 28 \end{bmatrix} = 146128.$$

Let's find the loop currents:

$$I_{11} = \frac{\Delta_1}{\Delta} = \frac{150928}{61448} = 2{,}456 \ (A),$$

$$I_{22} = \frac{\Delta_2}{\Delta} = \frac{171184}{61448} = 2{,}785 \ (A),$$

$$I_{33} = \frac{\Delta_2}{\Delta} = \frac{146128}{61448} = 2{,}378 \ (A).$$

All loop currents are positive, and therefore their actual directions coincide with the selected ones. Let's find the branch currents as the algebraic sum of the loop currents flowing in the branch.

$$I_1 = I_{11} - I_{33} = 2{,}456 - 2{,}378 = 0{,}078 \ (A),$$

$$I_2 = I_{11} = 2{,}456 \ (A),$$

$$I_3 = I_{22} - I_{44} = 2{,}786 - 2 = 0{,}786 \ (A),$$

$$I_4 = I_{33} - I_{44} = 2{,}378 - 2 = 0{,}378 \ (A),$$

$$I_5 = I_{22} - I_{11} = 2{,}786 - 2{,}456 = 0{,}33 \ (A),$$

$$I_6 = I_{22} - I_{33} = 2{,}786 - 2{,}378 = 0{,}408 \ (A),$$

$$I_7 = I_{22} = 2{,}786 \ (A).$$