# VINNITSA NATIONAL AGRARIAN UNIVERSITY

## Department of Electric Power Engineering, Electrical Engineering and Electromechanics
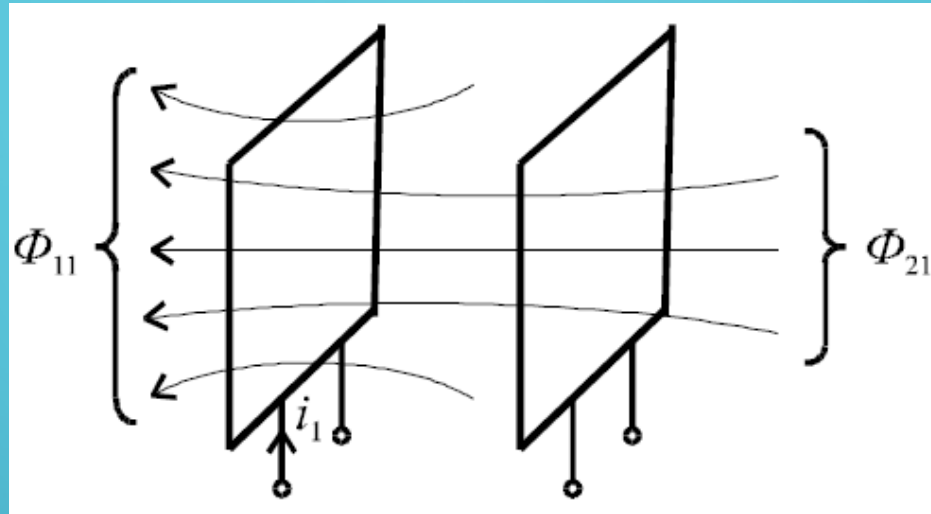


# CIRCUITS WITH MUTUAL INDUCTANCE

by Associate Professor V. Hraniak

# MUTUAL INDUCTANCE. ELECTROMOTIVE FORCE OF MUTUAL INDUCTANCE



The magnetic flux $\Phi$ in loop, which is formed by the electric current $I$ of this loop, is called the magnetic flux of self-induction (figure). Corresponding to this, the flux linkage of the loop (coils)

$$\psi = w\Phi$$

is called self-induction flux linkage, where w is the number of turns of the loop.

The inductance of the loop due to flux linkage and current can be determined as follows:

$$L = \frac{\psi}{i}.$$
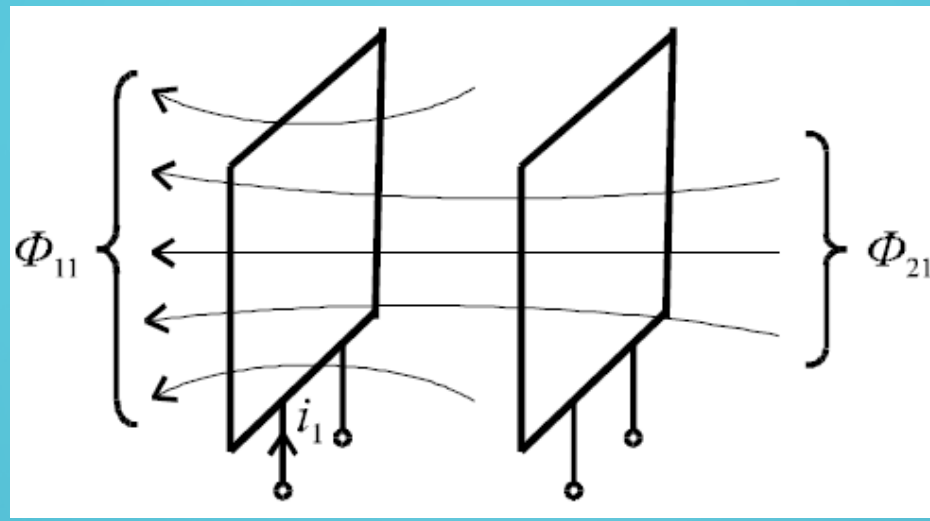
If the magnetic flux coupled to a given circuit is formed by the current of another circuit, then such a flux is called a mutual induction flux, and the corresponding flux linkage is a mutual induction flux linkage.

Consider the system of two coils shown in the figure, under the condition that the same flux is coupled to all turns of the coil w.

First, let the current $i_1 \neq 0$, and the current $i_2 = 0$. We denote by $\Phi_{11}$ – the flux connected to the first coil and formed by the current of the first coil, and $\Phi_{21}$ – the flux connected to the second coil and formed by the current of the first coil.

Corresponding flux linkages: $\psi_{11} = w_1\Phi_{11}$ and $\psi_{21} = w_2\Phi_{21}$.

Then the expression

$$\frac{\psi_{11}}{i_1} = L_1$$

is the inductance of the first coil, and the ratio

$$\frac{\psi_{21}}{i_1} = M_{21}$$

there is mutual inductance between the second and first coils.

If $i_1 = 0$, and $i_2 \neq 0$, then by analogy

$$\psi_{22} = w_2 \Phi_{22} \rightarrow \frac{\psi_{22}}{i_2} = L_2, \ a$$

$$\psi_{12} = w_1 \Phi_{12} \rightarrow \frac{\psi_{12}}{i_2} = M_{12},$$

respectively, the inductance of the second coil and the mutual inductance between the 1st and 2nd coils.

It can be shown that

$$M_{12} = M_{21} = M.$$

The mutual inductance M depends on the geometric dimensions of the coils, the number of turns w1 and w2, the magnetic permeability of the medium μ and the relative position of the coils. Like inductance, it is measured in henry (H).

If there are currents in both coils, then the complete flux linkages are determined by the formulas:

$$\begin{cases} \psi_1 = \psi_{11} \pm \psi_{12} = L_1 i_1 \pm M i_2; \\ \psi_2 = \psi_{22} \pm \psi_{21} = L_2 i_2 \pm M i_1. \end{cases}$$

Here, the "+" sign corresponds to the coordinated inclusion of the coils, when the self- and mutual-induction fluxes have the same direction, that is, they are added. The "-" sign corresponds to the opposite inclusion. The figure shows the conventional designations of agreed (a) and opposite (b) inclusions.
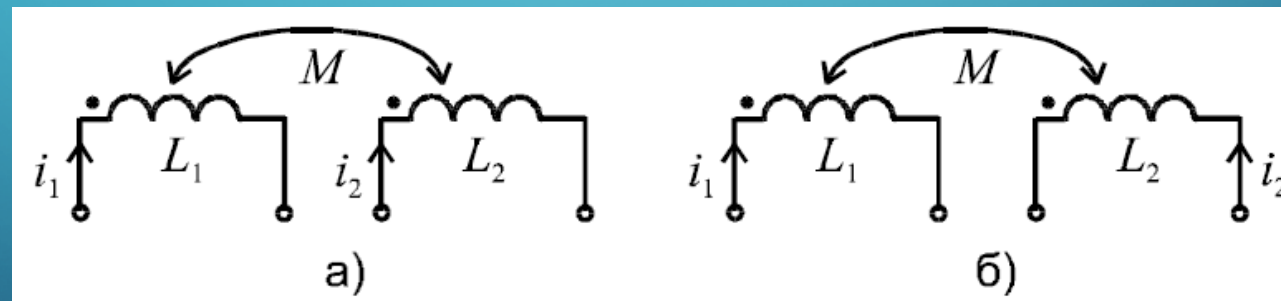


The symbols (•), (Δ), (*) on the electrical diagrams indicate the conditional beginnings of the coil. If the currents are oriented in the same way relative to such markings, then the inclusion is consistent.

Assume that the currents change with time. Then, according to the rule of electromagnetic induction, the EMF of the coils:

$$\begin{cases} e_1 = -\dfrac{d\psi_1}{dt} = -\dfrac{d}{dt}(L_1 i_1 \pm M i_2) = e_{L1} \pm e_{M1}; \\[2mm] e_2 = -\dfrac{d\psi_2}{dt} = -\dfrac{d}{dt}(L_2 i_2 \pm M i_1) = e_{L2} \pm e_{M2}, \end{cases}$$

where $e_{L1}$, $e_{M1}$, $e_{L2}$, $e_{M2}$ are EMFs of self-induction and mutual induction.

Electric voltages on the coils:

$$\begin{cases} u_1 = -e_1 = L_1 \dfrac{di_1}{dt} \pm M \dfrac{di_2}{dt} = u_{L1} \pm u_{M1}; \\[2mm] u_2 = -e_2 = L_2 \dfrac{di_2}{dt} \pm M \dfrac{di_1}{dt} = u_{L2} \pm u_{M2}. \end{cases}$$

If the currents are sinusoidal, then it is more convenient to use the complex form of recording

$$\underline{U}_1 = j\omega L_1 \underline{I}_1 \pm j\omega M \underline{I}_2 = \underline{U}_{L1} \pm \underline{U}_{M1},$$

where

$$\underline{U}_{M1} = j\omega M \underline{I}_2 = \underline{Z}_M \underline{I}_2.$$

The complex resistance of the mutual inductance can be determined as follows:

$$\underline{Z}_M = j\omega M = j\underline{X}_M.$$

In radio engineering and communication engineering, the concept of the coupling coefficient of two circuits is used

$$k_3 = \frac{\sqrt{\psi_{12}\psi_{21}}}{\sqrt{\psi_{11}\psi_{22}}} = \frac{\sqrt{Mi_1 \cdot Mi_2}}{\sqrt{L_1 i_1 \cdot L_2 i_2}} = \frac{M}{\sqrt{L_1 L_2}} < 1.$$

Devices in which k3 can be changed are called variometers
Practically, it can be a system of two connected flat coils, in which the moving coil has the ability to rotate relative to the stationary coil, that is, the coupling coefficient changes due to the change in the mutual inductance M.

# SERIES CONNECTION OF TWO INDUCTIVELY COUPLED COILS

Suppose that two real mutually inductive coils are connected in series and in agreement (figure)
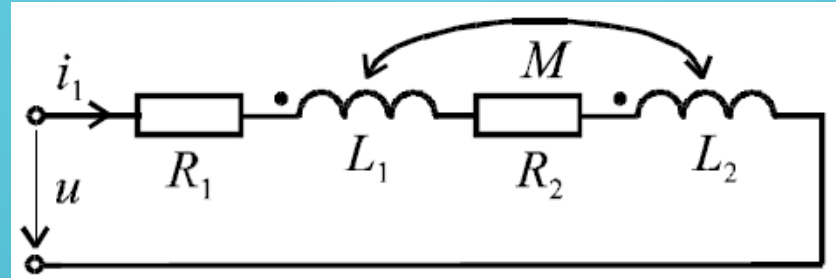


The equation of this circle in instantaneous form will look like this

$$u = R_1 i + L_1 \frac{di}{dt} + M \frac{di}{dt} + R_2 i + L_2 \frac{di}{dt} + M \frac{di}{dt},$$

or in a complex image for effective values of voltages and currents

$$\underline{U} = \underbrace{R_1 \underline{I} + j\omega L_1 \underline{I} + j\omega M \underline{I}}_{\underline{U}_{\kappa 1}} + \underbrace{R_2 \underline{I} + j\omega L_2 \underline{I} + j\omega M \underline{I}}_{\underline{U}_{\kappa 2}} = \underline{I}\underline{Z}_1 + \underline{I}\underline{Z}_2 = \underline{I}\underline{Z},$$
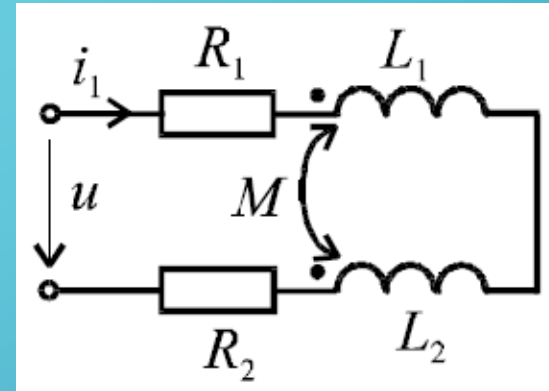
where

$$\underline{Z}_1 = R_1 + j\omega(L_1 + M) = R_1 + j\omega L_{1E}, \quad \underline{Z}_2 = R_2 + j\omega(L_2 + M) = R_2 + j\omega L_{2E};$$

# EXAMPLE

For a circuit with a series connection of two inductively coupled coils (figure), the voltage and circuit parameters are given. Determine the effective value of the current.

$U = 20$ V;
$R_1 = R_2 = 3$ Ohm;
$\omega L_1 = \omega L_2 = 10$ Ohm;
$\omega M = 6$ Ohm.



The solution

In this case, we have counter-inclusion of the coils.

Fule circuit resistance:

$$\underline{Z} = (R_1 + R_2) + j(\omega L_1 + \omega L_2 - 2\omega M) = (3+3) + j(10+10-12) = 6 + j8 = 10e^{j53°} \; Ohm.$$

Complex effective current value

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{20}{10e^{j53°}} = 2e^{-j53°} \; A.$$

Effective current value $I = 2$ A.

# PECULIARITIES OF CONSTRUCTING EQUATIONS FOR A COMPLEX CIRCUIT WITH MUTUAL INDUCTANCE

Kirchhoff's equation and the method of loop currents are used for such circles. The method of nodal voltages, the method of the equivalent generator, the method of equivalent transformations in the general case cannot be applied, since they do not take into account inductive connections. If we go from the valid circuit to an equivalent circuit without inductive connections, then all methods can be applied. The transition to an equivalent circuit without inductive connections is possible under the conditions of an equivalent transformation.
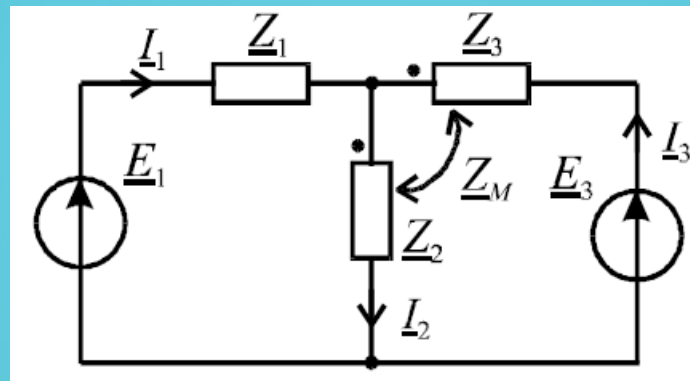
Let's formulate the rules for compiling equations according to Kirchhoff's laws for circles with mutual inductive connections. Consider this on the example of a two-circuit circuit (figure). We arbitrarily specify the directions of the branch currents. Uncoordinated switching of the coils corresponds to these directions.

Compiling the equation according to Kirchhoff's first rule has no peculiarities
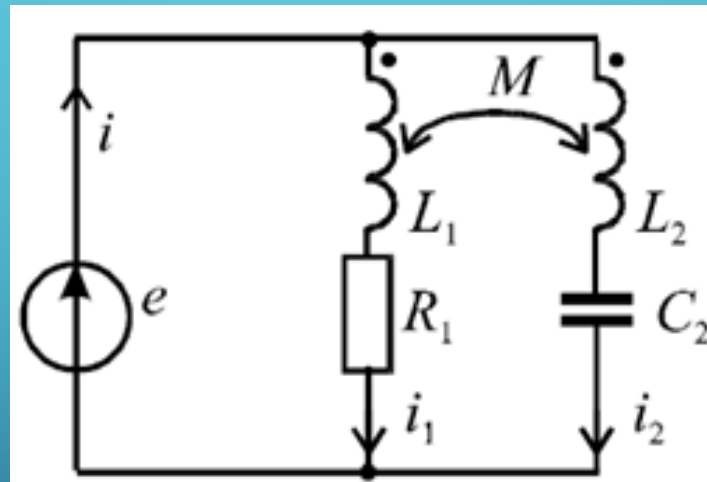
$$I_1 - I_2 + I_3 = 0.$$

Compilation of contour equations according to Kirchhoff's second rule has the peculiarity that when bypassing an inductively coupled coil, the voltages introduced by mutual induction are written with the same sign as the voltage from the coil's own current, if they are turned on in concert, and with with the opposite sign if they are included oppositely:

$$\underline{I}_1\underline{Z}_1 + \underline{I}_2\underline{Z}_2 - \underline{I}_3\underline{Z}_M = \underline{E}_1,$$

$$\underline{I}_3\underline{Z}_3 - \underline{I}_2\underline{Z}_M + \underline{I}_2\underline{Z}_2 - \underline{I}_3\underline{Z}_M = \underline{E}_3.$$

# EXAMPLE
# (INDIVIDUAL WORK)

Compile a system of equations according to Kirchhoff's rules to calculate the proposed electric circuit

# THANK FOR YOUR ATTENTION!