VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of Electric Power Engineering, Electrical Engineering and Electromechanics

CALCULATION OF COMPLEX SINUSOIDAL CIRCLES (Kirchhoff's rules and the method of loop currents)

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METHOD OF LOOP CURRENTS

It is advisable to use the method of loop currents if the number of equations according to Kirchhoff's second rule is less than the number of equations according to the first rule.

The essence of the method is that instead of currents in the branches, new variables are introduced - closed currents that conditionally pass through the branches of independent loops. These currents are called loop currents and their number is less than the number of branch currents.

The convenience of this method also lies in the fact that its system of equations has a unified form for any circle scheme.

In the general case, the system of equations for n loops has the form:

$$\underline{Z}_{11}\underline{J}_1+\underline{Z}_{12}\underline{J}_2+\underline{Z}_{13}\underline{J}_3+\ldots+\underline{Z}_{1n}\underline{J}_n=\underline{E}_{11},$$

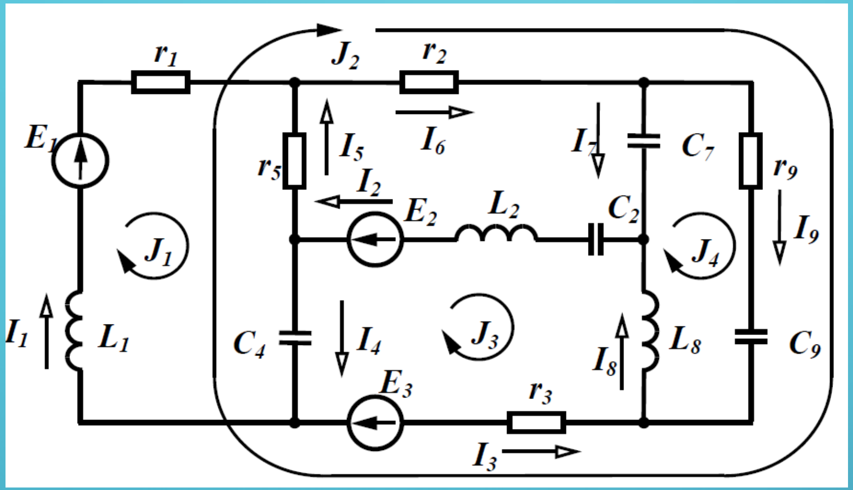
$$\underline{Z}_{21}\underline{J}_1+\underline{Z}_{22}\underline{J}_2+\underline{Z}_{23}\underline{J}_3+\ldots+\underline{Z}_{2n}\underline{J}_n=\underline{E}_{22},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\underline{Z}_{n1}\underline{J}_1+\underline{Z}_{n2}\underline{J}_2+\underline{Z}_{n3}\underline{J}_3+\ldots+\underline{Z}_{nn}\underline{J}_n=\underline{E}_{nn}.$$

EXAMPLE

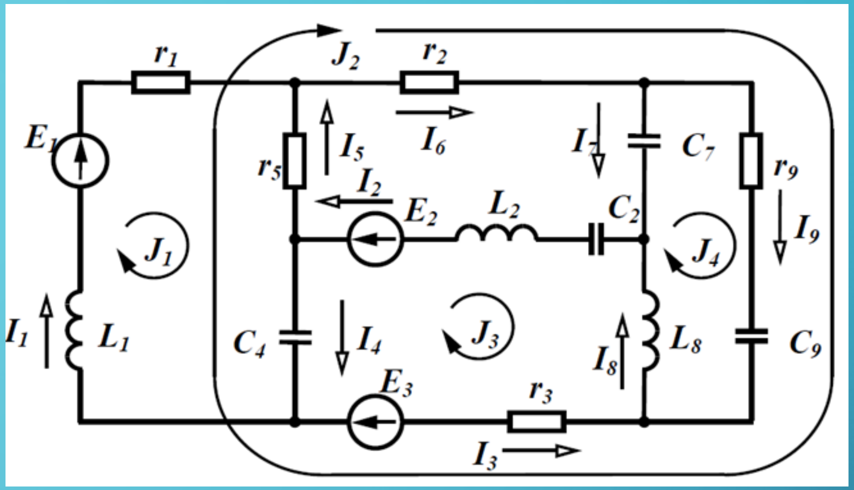
For the circle shown in the figure, complete a system of equations using the method of loop currents.



There are four independent loops in the circle. Let's set the direction of the loop currents as shown in the figure, and write the equation for four loops in general form:

$$
\begin{cases}
\underline{Z}_{11}\underline{J}_1 + \underline{Z}_{12}\underline{J}_2 + \underline{Z}_{13}\underline{J}_3 + \underline{Z}_{14}\underline{J}_4 = \underline{E}_{11}, \\
\underline{Z}_{21}\underline{J}_1 + \underline{Z}_{22}\underline{J}_2 + \underline{Z}_{23}\underline{J}_3 + \underline{Z}_{24}\underline{J}_4 = \underline{E}_{22}, \\
\underline{Z}_{31}\underline{J}_1 + \underline{Z}_{32}\underline{J}_2 + \underline{Z}_{33}\underline{J}_3 + \underline{Z}_{34}\underline{J}_4 = \underline{E}_{33}, \\
\underline{Z}_{41}\underline{J}_1 + \underline{Z}_{42}\underline{J}_2 + \underline{Z}_{43}\underline{J}_3 + \underline{Z}_{44}\underline{J}_4 = \underline{E}_{44}.
\end{cases}
$$

Let determine all the coefficients:



$$\underline{Z}_{11} = r_1 + r_5 + j\omega L_1 - j\frac{1}{\omega C_4}.$$

$$\underline{Z}_{22} = r_3 + r_5 + r_6 + r_9 - j\frac{1}{\omega C_4} - j\frac{1}{\omega C_9},$$

$$\underline{Z}_{33} = r_3 + j\omega L_2 + j\omega L_8 - j\frac{1}{\omega C_2} - j\frac{1}{\omega C_4},$$

$$\underline{Z}_{44} = r_9 - j\frac{1}{\omega C_7} - j\frac{1}{\omega C_9} + j\omega L_8.$$

$$\underline{Z}_{12} = \underline{Z}_{21} = -\left(r_5 - j\frac{1}{\omega C_4}\right).$$

$$\underline{Z}_{13} = \underline{Z}_{31} = -\left(-j\frac{1}{\omega C_4}\right).$$

$$\underline{Z}_{14} = \underline{Z}_{41} = 0.$$

$$\underline{Z}_{23} = \underline{Z}_{32} = r_3 - j\frac{1}{\omega C_4},$$

$$\underline{Z}_{24} = \underline{Z}_{42} = r_9 - j\frac{1}{\omega C_9},$$

$$\underline{Z}_{34} = \underline{Z}_{43} = -(j\omega L_8),$$

$$\underline{E}_{11} = \underline{E}_1,\ \underline{E}_{22} = \underline{E}_3,\ \underline{E}_{33} = -\underline{E}_2 + \underline{E}_3,\ \underline{E}_{44} = 0.$$

THE METHOD OF KIRCHHOFF'S RULES

In most tasks of electric circuit analysis, it is possible to choose one of the methods that significantly reduce the rank of the system of equations.

However, there are circles for which calculation is possible only by direct use of Kirchhoff's laws. These are circuits with dependent (controlled) power sources.

Kirchhoff's first rule

$$\sum_{k=0}^{n} \underrightarrow{I_k} = 0$$

Kirchhoff's second rule

$$\sum_{k=0}^{n} \underrightarrow{E_k} = \sum_{k=0}^{m} \underrightarrow{I_k} \cdot \underrightarrow{Z_k}$$

EXAMPLE

For the circuit in the figure, find all the currents.

$$e_1(t) = 100 \cdot \sin\left(\omega t - 20^\circ\right) V$$

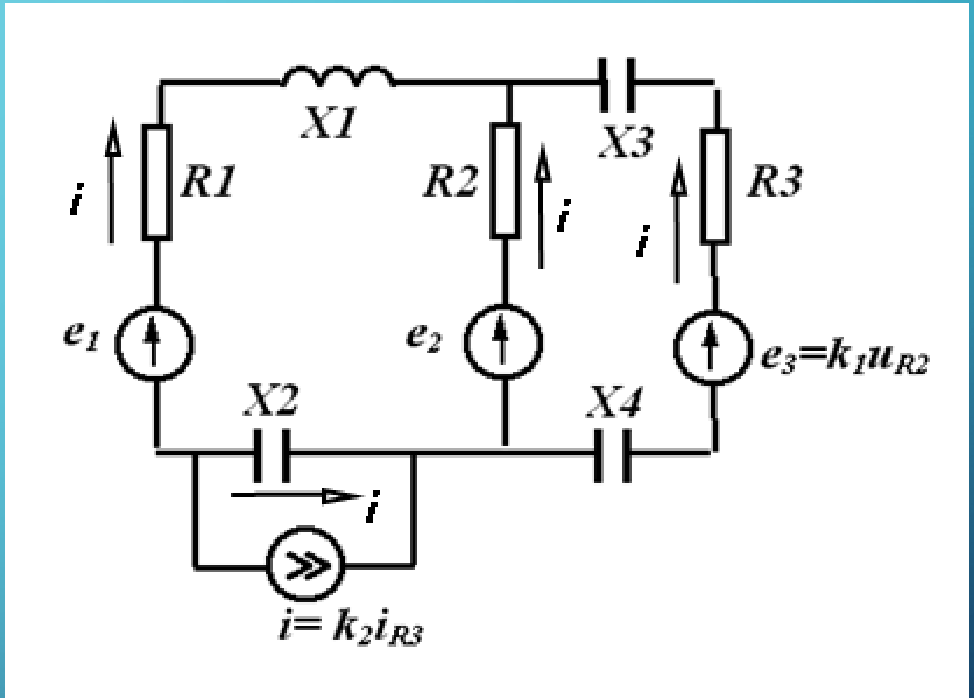
$$e_2(t) = 150 \cdot \sin\left(\omega t + 30^\circ\right) V$$

$R_1 = 6\ Ohm \qquad X_2 = 6\ Ohm$

$R_2 = 10\ Ohm \qquad X_3 = 4\ Ohm$

$R_3 = 8\ Ohm \qquad X_4 = 12\ Ohm$

$X_1 = 9\ Ohm \qquad k_1 = 0,8 \qquad k_2 = 0,5$



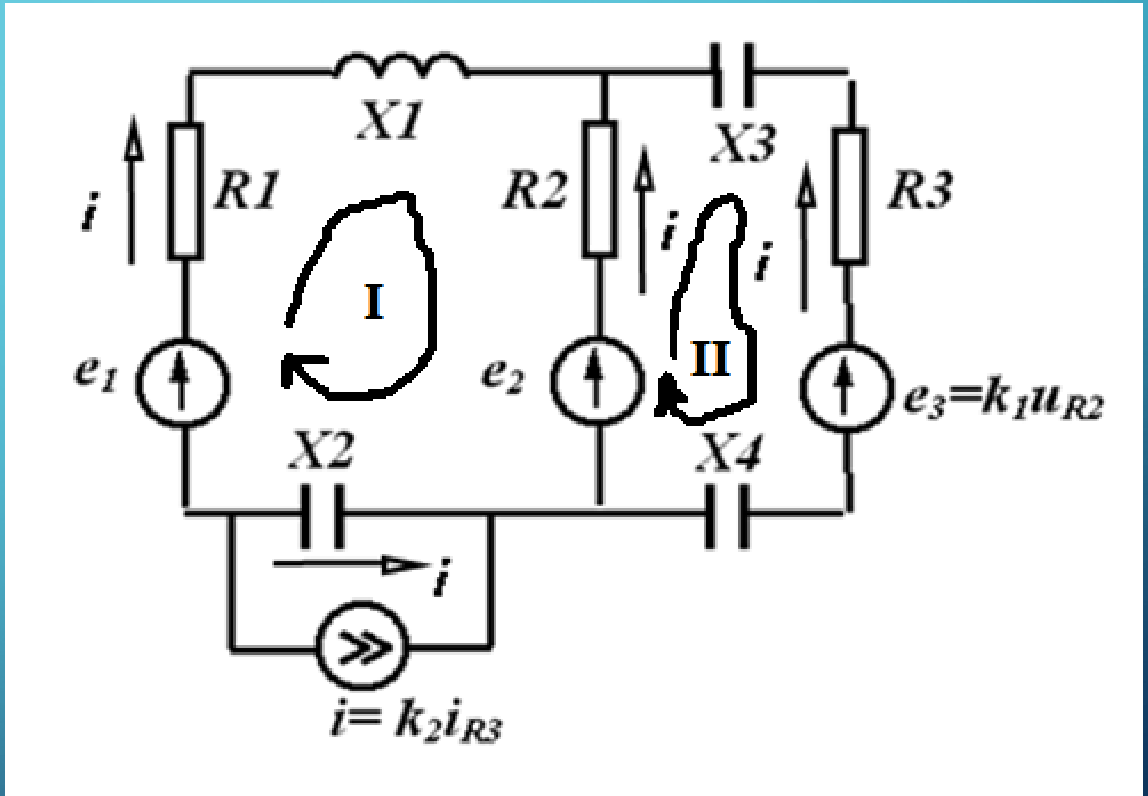
Let's write the expressions for the complete complex resistances of the circuits, assuming that the circuit number coincides with the current number

$$\underline{Z}_1 = R_1 + jX_1$$

$$\underline{Z}_2 = R_2$$

$$\underline{Z}_3 = R_3 - j(X_3 + X_4)$$

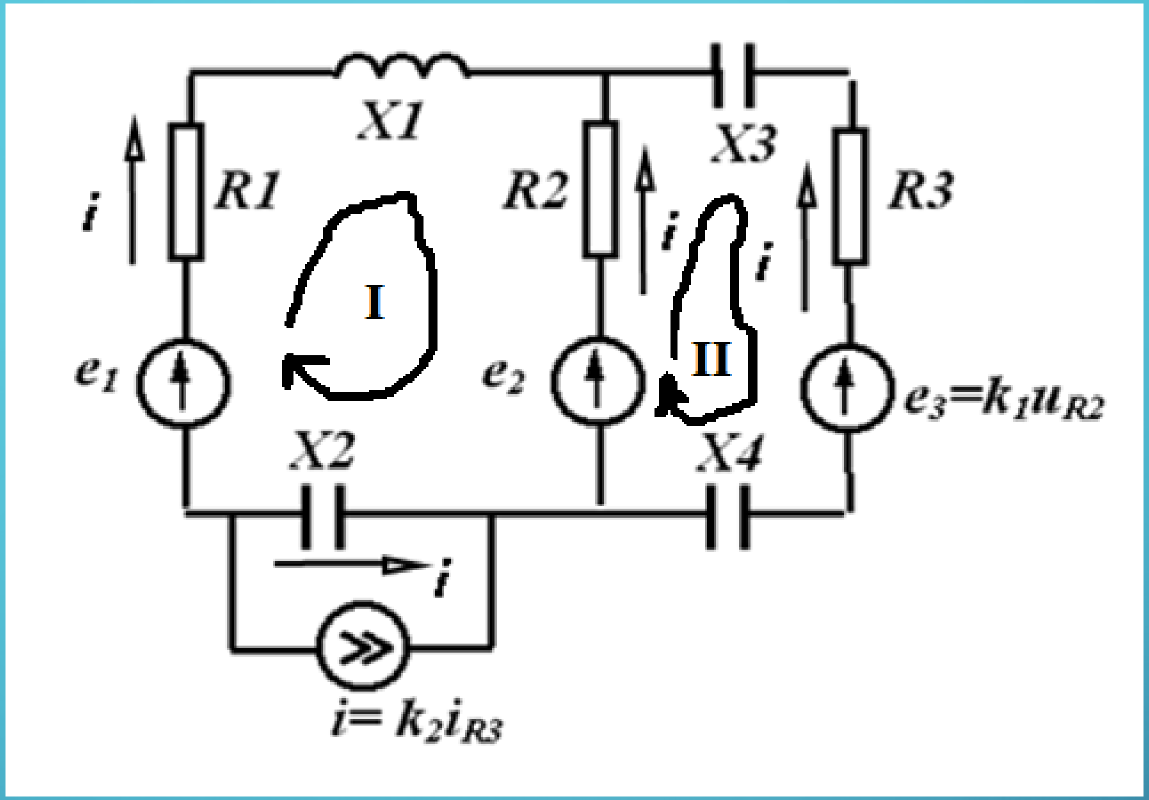
$$\underline{Z}_4 = -jX_2$$



System of equations

$$\begin{cases} \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0, \\ \underline{I} + \underline{I}_4 - \underline{I}_2 - \underline{I}_3 = 0, \\ \underline{I}_1\underline{Z}_1 - \underline{I}_2\underline{Z}_2 - \underline{I}_4\underline{Z}_4 = \underline{E}_1 - \underline{E}_2, \\ \underline{I}_2\underline{Z}_2 - \underline{I}_3\underline{Z}_3 = \underline{E}_2 - \underline{E}_3. \end{cases}$$

$$\underline{E}_3 = k_1\underline{I}_2\underline{Z}_2 \qquad \underline{I} = k_2\underline{I}_3$$



Taking into account dependent power sources

$$\begin{cases} \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0, \\ \underline{I}_4 - \underline{I}_2 + \underline{I}_3 (k_2 - 1) = 0, \\ \underline{I}_1 \underline{Z}_1 - \underline{I}_2 \underline{Z}_2 - \underline{I}_4 \underline{Z}_4 = \underline{E}_1 - \underline{E}_2, \\ \underline{I}_2 \underline{Z}_2 (k_1 + 1) - \underline{I}_3 \underline{Z}_3 = \underline{E}_2. \end{cases}$$

We will complete all calculations in the MathCAD.
Let's enter the input data.

$Em1 := 100 \quad \beta1 := -20deg \quad Em2 := 150 \quad \beta2 := 30deg$

$r1 := 6 \quad r2 := 10 \quad r3 := 8 \quad x1 := 9 \quad x2 := 6$

$x3 := 4 \quad x4 := 12 \quad k1 := 0.8 \quad k2 := 0.5$

$$E1 := \frac{Em1}{\sqrt{2}} \cdot e^{i \cdot \beta1} \qquad E2 := \frac{Em2}{\sqrt{2}} \cdot e^{i \cdot \beta2}$$

$$E1 = 66.446 - 24.184i \qquad E2 = 91.856 + 53.033i$$

$$Z1 := r1 + i \cdot x1 \quad Z2 := r2 \quad Z3 := r3 - i \cdot (x3 + x4) \quad Z4 := -(i \cdot x2)$$

Based on the system, we enter into MathCAD the matrix of coefficients for unknown currents and the vector of the right parts, after which we find the complex values of the currents.

$$A := \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -1 & k2-1 & 1 \\ Z1 & -Z2 & 0 & -Z4 \\ 0 & Z2\cdot(k1+1) & -Z3 & 0 \end{bmatrix} \qquad B := \begin{pmatrix} 0 \\ 0 \\ E1-E2 \\ E2 \end{pmatrix}$$

$$J := \text{lsolve}(A,B) \qquad I1 := J_0 \qquad I2 := J_1 \qquad I3 := J_2 \qquad I4 := J_3$$

$$I1 = -0.682 - 5.038i \qquad |I1| = 5.084 \qquad \arg(I1) = -97.709\,\text{deg}$$

$$I2 = 3.549 + 5.354i \qquad |I2| = 6.423 \qquad \arg(I2) = 56.464\,\text{deg}$$

$$I3 = -2.867 - 0.316i \qquad |I3| = 2.884 \qquad \arg(I3) = -173.717\,\text{deg}$$

$$I4 = 2.115 + 5.196i \qquad |I4| = 5.61 \qquad \arg(I4) = 67.849\,\text{deg}$$

Let implement calculations in MathCAD..

$E3 := k1 \cdot I2 \cdot Z2 \quad E3 = 28.389 + 42.832i \quad |E3| = 51.386 \quad arg(E3) = 56.464\, deg$

$I := k2 \cdot I3 \quad I = -1.433 - 0.158i \quad |I| = 1.442 \quad arg(I) = -173.717\, deg$

$Ui := I4 \cdot Z4 \quad Ui = 31.177 - 12.692i \quad |Ui| = 33.662 \quad arg(Ui) = -22.151\, deg$

$$Sd := E1 \cdot \overline{I1} + E2 \cdot \overline{I2} + E3 \cdot \overline{I3} - Ui \cdot \overline{I}$$

$$Ssp := (|I1|)^2 \cdot Z1 + (|I2|)^2 \cdot Z2 + (|I3|)^2 \cdot Z3 + (|I4|)^2 \cdot Z4$$

$Sd = 634.217 - 89.257i \qquad Ssp = 634.217 - 89.257i$

THANK FOR YOUR ATTENTION!