VINNITSA NATIONAL AGRARIAN UNIVERSITY

Department of Electric Power Engineering, Electrical Engineering and Electromechanics

METHOD OF NODAL POTENTIALS

by Associate Professor V. Hraniak

UNIVERSAL SYSTEM OF EQUATIONS

$$\begin{cases} \phi_1 Y_{11} - \phi_2 Y_{12} - \ldots - \phi_n Y_{1n} = I_{11}, \\ -\phi_1 Y_{21} + \phi_2 Y_{22} - \ldots - \phi_n Y_{2n} = I_{22}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -\phi_1 Y_{n1} - \phi_2 Y_{n2} - \ldots + \phi_n Y_{nn} = I_{nn}, \end{cases}$$

where Y_{ii} is the own conductance of the i-th node, defined as the sum of the conductances of the branches connected to this node; Y_{ij} is the joint conductivity of the i-th and j-th nodes, defined as the sum of the conductances of the branches simultaneously connected to the i-th and j-th nodes; Iii is the own current of the i-th node, defined as the sum of the products of the EMF on the conductivity of the active loops connected to the i-th node (in the case when the EMF is directed to the i-th node, its direction is considered positive, in the other case - negative).

ALGORITHM FOR SOLVING AN ELECTRIC CIRCUIT BY THE METHOD OF NODAL POTENTIALS

1. Take the potential of one of the nodes as zero (ground it).

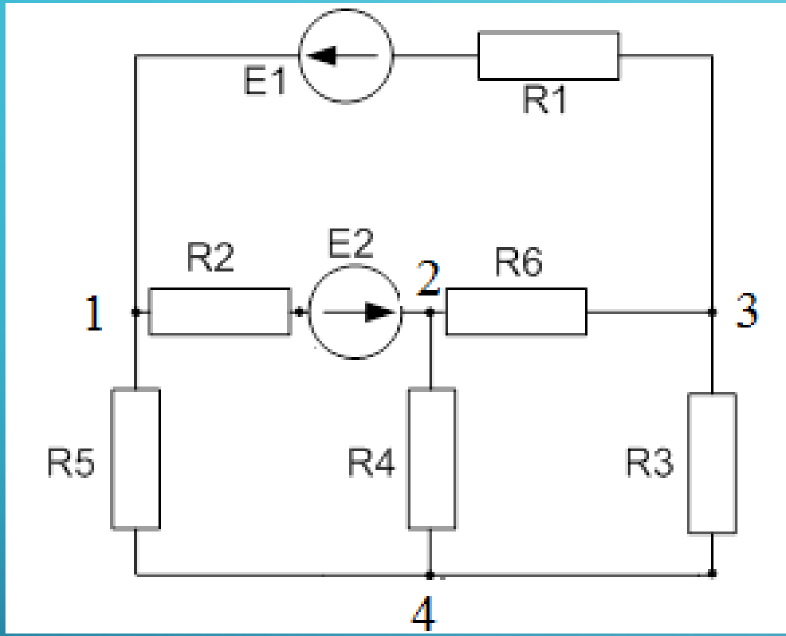
2. Compile equations using the method of nodal potentials. The number of equations is equal to the number of remaining nodes.

3. Using any calculation method, solve the system of equations and determine the potentials of the nodes.

4. Based on the found nodal potentials, determine the currents in the branches of the circuit.

EXAMPLE

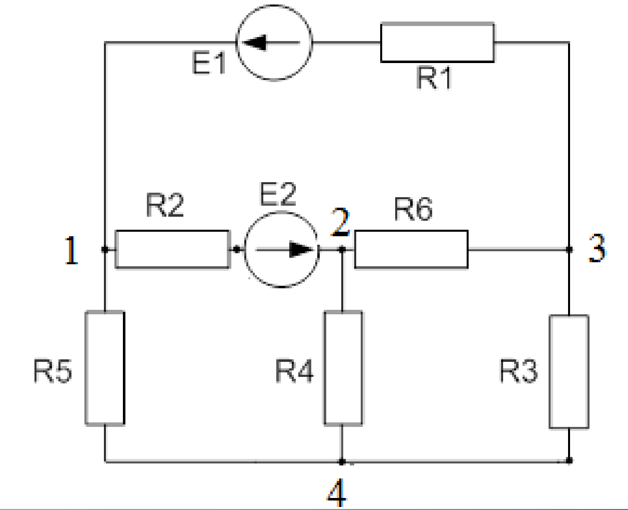
For the circle depicted in the figure, make a system of equations using the method of nodal potentials



Ground node 4 and write down the system of equations using the nodal potential method in general form:

$$\begin{cases} \phi_1 Y_{11} - \phi_2 Y_{12} - \phi_3 Y_{13} = I_{11}, \\ -\phi_1 Y_{21} + \phi_2 Y_{22} - \phi_2 Y_{23} = I_{22}, \\ -\phi_1 Y_{31} - \phi_2 Y_{32} + \phi_3 Y_{33} = I_{33}. \end{cases}$$

Let's determine the own and joint conductances of nodes and node currents.



$$Y_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5},$$

$$Y_{22} = \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_6},$$

$$Y_{33} = \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_6},$$

$$Y_{12} = Y_{21} = \frac{1}{R_2},$$

$$Y_{13} = Y_{31} = \frac{1}{R_1},$$

$$Y_{23} = Y_{32} = \frac{1}{R_6},$$

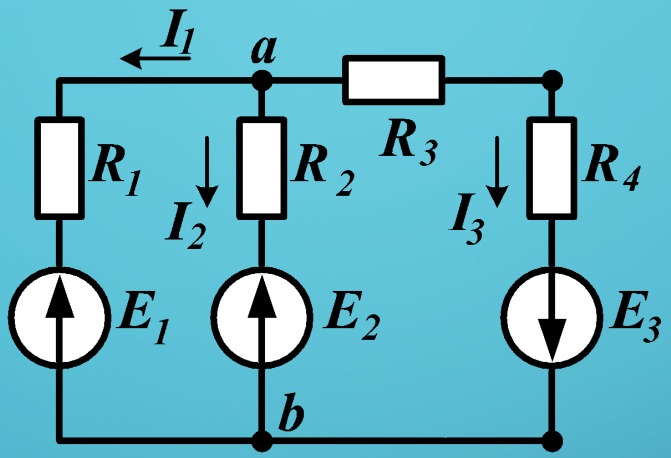
$$I_{11} = E_1 \frac{1}{R_1} - E_2 \frac{1}{R_2},$$

$$I_{22} = E_2 \frac{1}{R_2},$$

$$I_{33} = -E_1 \frac{1}{R_i}.$$

EXAMPLE OF CALCULATION

For the circle depicted in the figure, make a system of equations using the method of nodal potentials

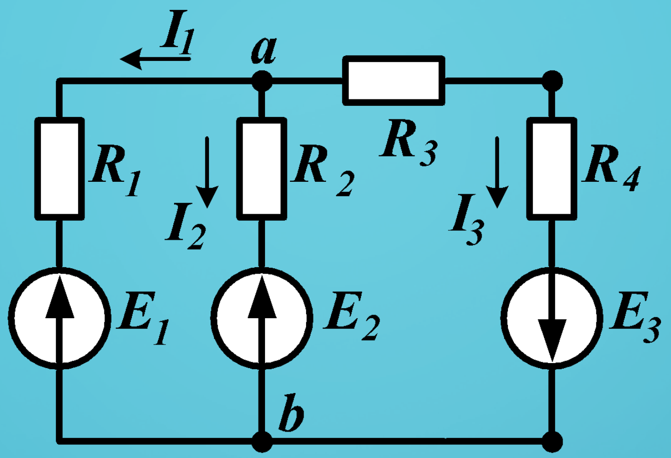


$$\phi_a = \phi_1 \qquad \phi_b = 0$$

$$\phi_1 Y_{11} = I_{11}$$

$$Y_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3 + R_4} \qquad I_{11} = \frac{E_1}{R_1} + \frac{E_2}{R_2} - \frac{E_3}{R_3 + R_4}$$

CALCULATION OF CURRENTS

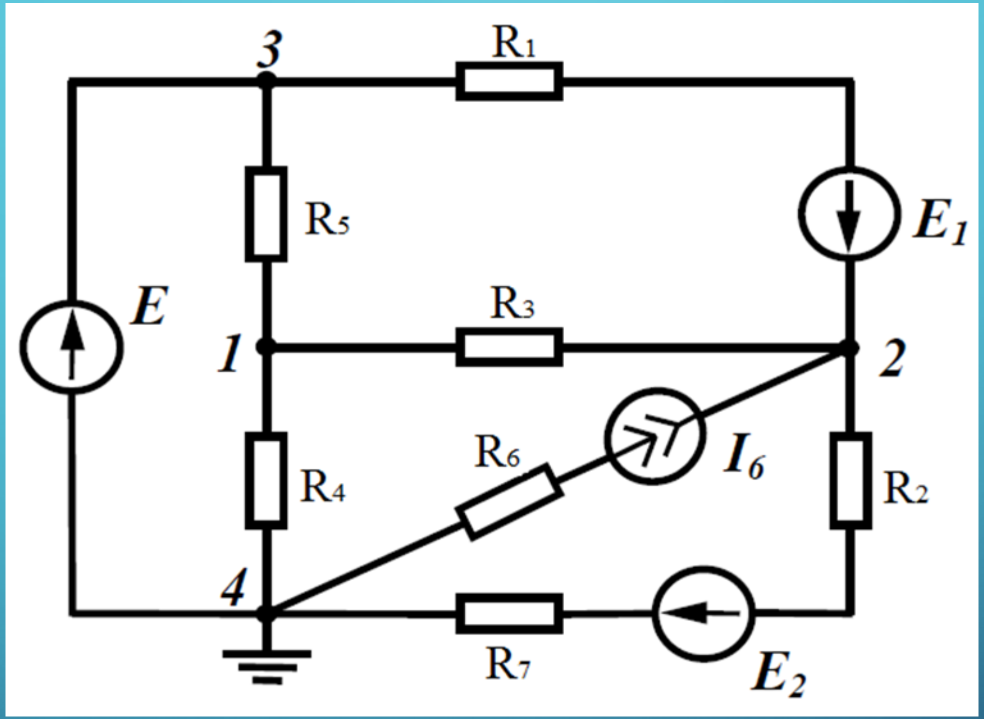


$$I_1 = \frac{\phi_1 - E_1}{R_1}$$

$$I_2 = \frac{\phi_1 - E_2}{R_2}$$

$$I_3 = \frac{\phi_1 + E_3}{R_3 + R_4}$$

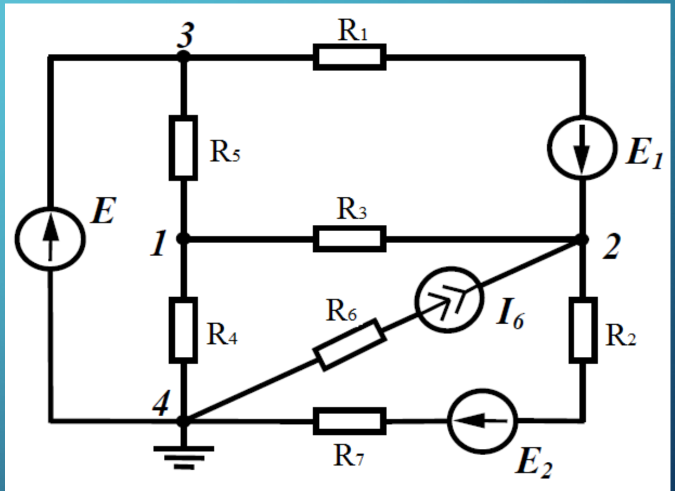
PECULIARITIES OF THE NODAL POTENTIAL METHOD IN CASE WHEN THERE ARE IDEAL VOLTAGE AND CURRENT SOURCES IN ELECTRICAL CIRCUIT

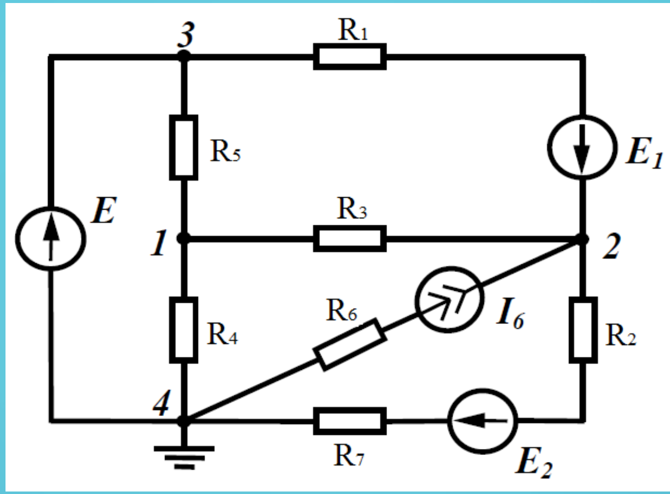


If there is a branch between two nodes containing an ideal EMF and does not contain passive elements, then when grounding one of these nodes (for example, node 4), to wit assuming $\varphi 4 = 0$, it is easy to find the potential of node 3, since $\varphi 3 - \varphi 4 = E$, and from it $\varphi 3 = E$.

Thus, the number of unknown potentials has become less by one, and for this scheme it is necessary to compose a system of only two equations, leaving the component with the known potential φ3 in the left part.

$$\begin{cases} \varphi_1 Y_{11} - \varphi_2 Y_{12} - \varphi_3 Y_{13} = I_{11}, \\ -\varphi_2 Y_{21} + \varphi_2 Y_{22} - \varphi_2 Y_{23} = I_{22}. \end{cases}$$





When finding own and joint conductances in this system, it should be taken into account that branch 6 contains an ideal current source. As indicated earlier, the current in this circuit is equal to the current of the current source I6, and the internal resistance of the ideal current source is infinitely large, so the conductivity of this circuit Y6 = 0.

In this case, it is convenient to number the nodes in such a way that the nodes that are adjacent to the branch with only one ideal EMF are marked with the last numbers in order.

Taking this into account, we write down the coefficients of the left part of the equation.

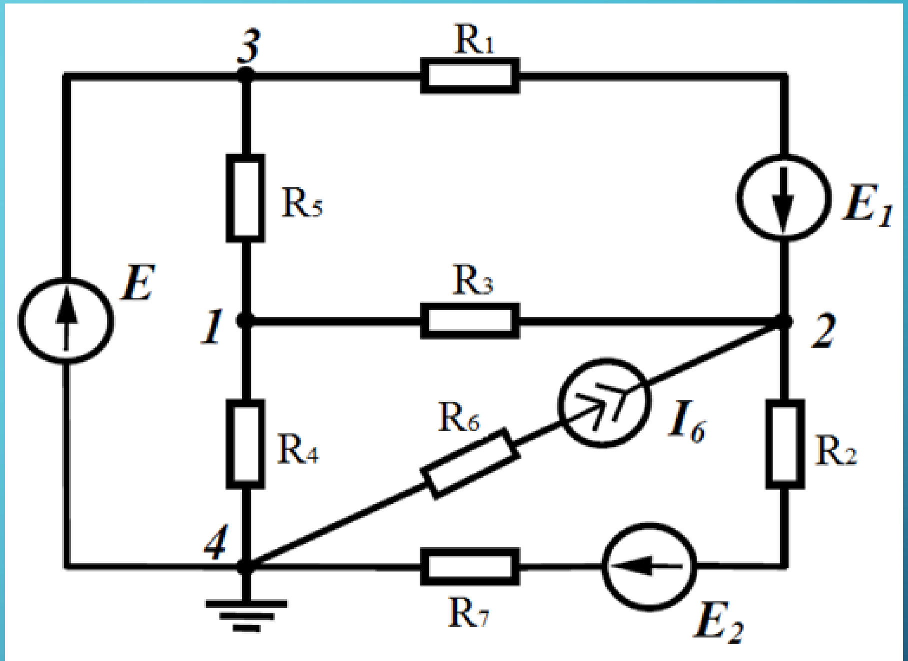
$$Y_{11} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5},$$

$$Y_{22} = \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2 + R_7},$$

$$Y_{12} = Y_{21} = \frac{1}{R_3},$$

$$Y_{13} = Y_{31} = \frac{1}{R_5},$$

$$Y_{23} = Y_{32} = \frac{1}{R_1},$$



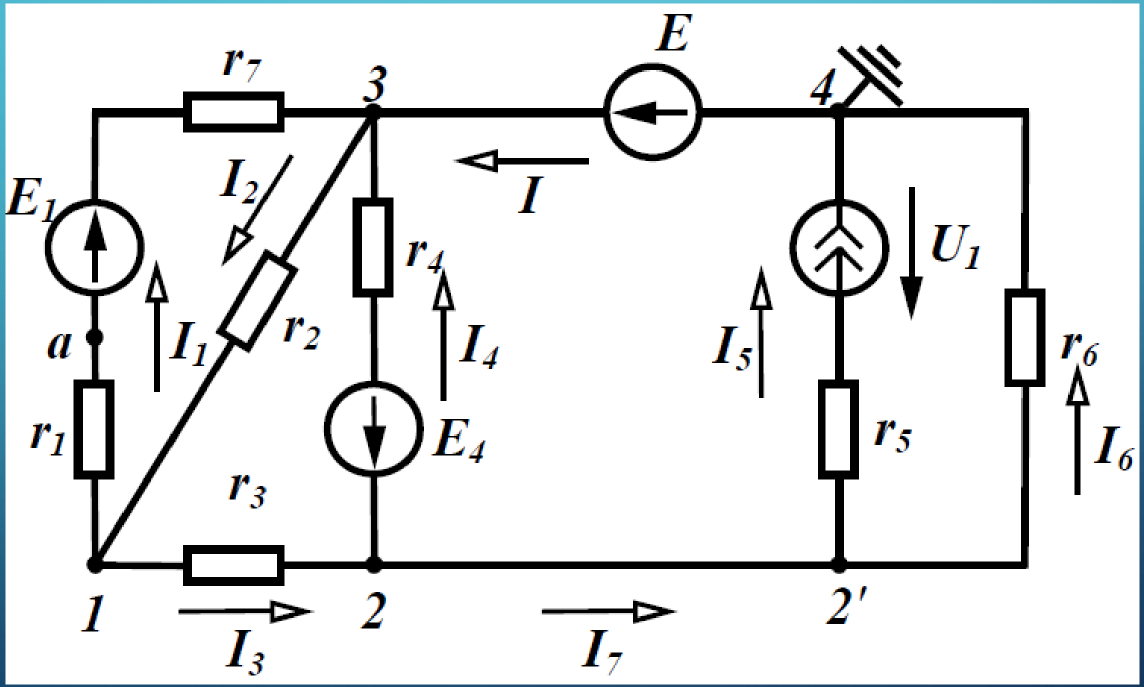
$$I_{11} = 0,$$

$$I_{22} = E_1 \frac{1}{R_1} - E_2 \frac{1}{R_2 + R_7} + I_6.$$

EXAMPLE

For the circuit with parameters given in the figure, find the currents in all branches of the circuit using the method of nodal potentials.

$E = 100$ V, $E_1 = 100$ V, $E_4 = 30$ V, $I_5 = 7{,}5$ A,
$r_1 = 4$ Ohm, $r_2 = 5$ Ohm, $r_3 = 10$ Ohm,
$r_4 = 4$ Ohm, $r_5 = 16$ Ohm, $r_6 = 20$ Ом, $r_7 = 6$ Ohm



THANK FOR YOUR ATTENTION!